A STUDY ON SPECTRAL AND STATISTICAL ANALYSIS OF

NOISY SIGNALS FOR PHYSICAL PROCESSES

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Abstract:

In this study, it is considered a sinusoidal signal with its third harmonic as well as the fundamental frequency adding the noise term, which is produced using a Gaussian random number generator. Hence spectral and statistical properties of this signal are examined and Signal-to-Noise Ratio (SNR) is computed comparing with the probability density function of the noisy signal.

Keywords: *Power Spectrum, Statistical Parameters, Harmonics, Gaussian Noise.*

1. INTRODUCTION

In most physical processes, to define the system properties and its behavior, there are needs to various signal characteristics. For this purpose, different approaches are used in the framework of the signal analysis techniques. But, in this manner, the most popular one is the spectral analysis approach to extract the useful information in the frequency domain [1, 2]. Hence the signal content is represented with these frequency components as a result of the spectral analysis. In addition to this approach, the other one is the statistical analysis approach. With this way, the statistical properties are represented by its parametric values. These parametric values specially mean value, standard deviation, skewness and kurtosis. The mean value and standard deviation measure the central tendency and deviation from the central value respectively. These are the fundamental parameters. The last two are, namely parameters of the skewness and kurtosis, are related to the measure of the symmetry and the sharpness of the peak of the probability density function considered for the noise term respectively. Hence the various complex signals, which are considered by the noise term and harmonics, used to simulate the physical system behavior are analyzed to extract the useful information. In this sense, this study reflects the various spectral and statistical aspects of these kinds of the complex signals as a powerful hybrid approach.

2. SPECTRAL DOMAIN ANALYSIS

A common approach for extracting the information about the frequency features of a random signal is to transform the signal to the frequency domain by computing the discrete Fourier transform. For a block of data of length N samples the transform at frequency $m\Delta f$ is given by

$$X(m\Delta f) = \sum_{k=0}^{N-1} x(k\Delta t) \exp[- j2\pi km / N] \quad (1)$$

where Δf is the frequency resolution and Δt is the data-sampling interval. The auto-power spectral density (APSD) of $x(t)$ is estimated as

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$$S_{xx}(f) = \frac{1}{N} \left| X(m\Delta f) \right|^2 , \; f = m\Delta f \quad (2)$$

The cross power spectral density (CPSD) between $x(t)$ and $y(t)$ is similarly estimated. The statistical accuracy of the estimate in eq. (2) increases as the number of data points or the number of blocks of data increases [3].

3. BASIC STATISTICAL PARAMETERS FOR DATA ANALYSIS

Several statistical parameters, calculated in the time domain, are generally used to define average properties of machinery data. The two basic parameters are the mean value, μ and the standard deviation, σ. For a given data set $\{x_i\}$ these are defined as follows:

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i , \quad (3)$$

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2} , \quad (4)$$

where N is the number of the data points.

For the Gaussian (normal) probability distribution, two parameters that reflect the departure from the normal distribution are skewness (s) and kurtosis (k). These are calculated as follows:

$$s = \frac{\left[\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^3 \right]}{\sigma^3} , \quad (5)$$

$$k = \frac{\left[1/N \sum_{i=1}^{N} (x_i - \mu)^4 \right]}{\sigma^4} \quad (6)$$

For a perfect normal distribution, s is equal to zero. A negative value is due to skewness towards lower values while a positive value indicates non-symmetry towards higher values. For small data sets, one often gets values that differ from zero. The kurtosis or flatness k is very close to unity for a normal distribution. These statistical parameters may be used to perform a quick check of the changes in the statistical behavior of a signal [4, 5].

4. SIMULATION OF THE NOISY SIGNAL

To simulate the noisy signal, considering the sampling frequency at 1 kHz, it is used sinusoidal components at the 50 Hz and 150 Hz as a third harmonic of the fundamental frequency value; hence deterministic signal part can be represented by these two components. After that, deviation adding a noise term in the normal distribution function

with the unit standard deviation and zero mean value like $N(0, 1)$, noisy signal can be defined. In this manner, the simulated noisy signal with its harmonic can be shown as below.
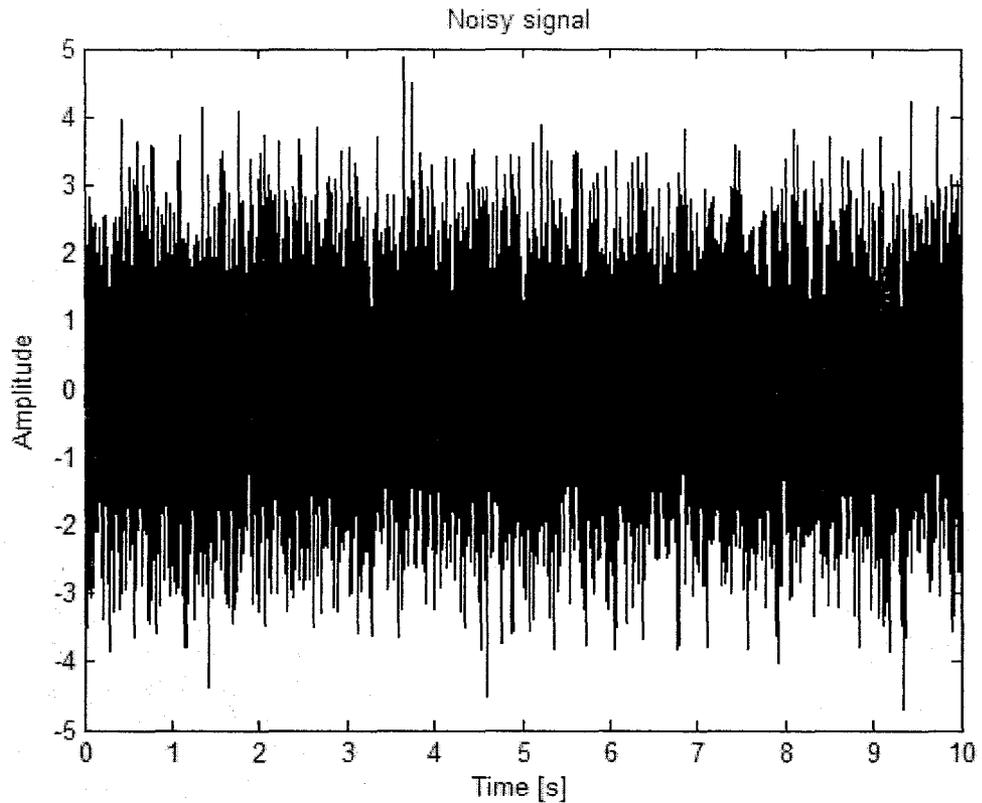

Figure 1. Simulated noisy signal with the additive noise term and third harmonic

In this signal, the noise term is highly dominant, therefore the useful information, which are represented at the 50 Hz and 150 Hz, is hidden information. The main goal of this study is to extract this useful information and show the statistical properties of the noisy signal according to the noise level.

5. APPLICATION

If the power spectrum, which is defined in the section 2, is applied to the simulated signal, the sinusoidal components of the noisy signal can be extracted. However, when the noise part is dominant over the deterministic parts, to show them at the specific frequencies is very difficult. For this reason, to prevent this situation semi-logarithmic scale is used. As a result the simulated data like Figure 1 can be represented by the following figure 2.
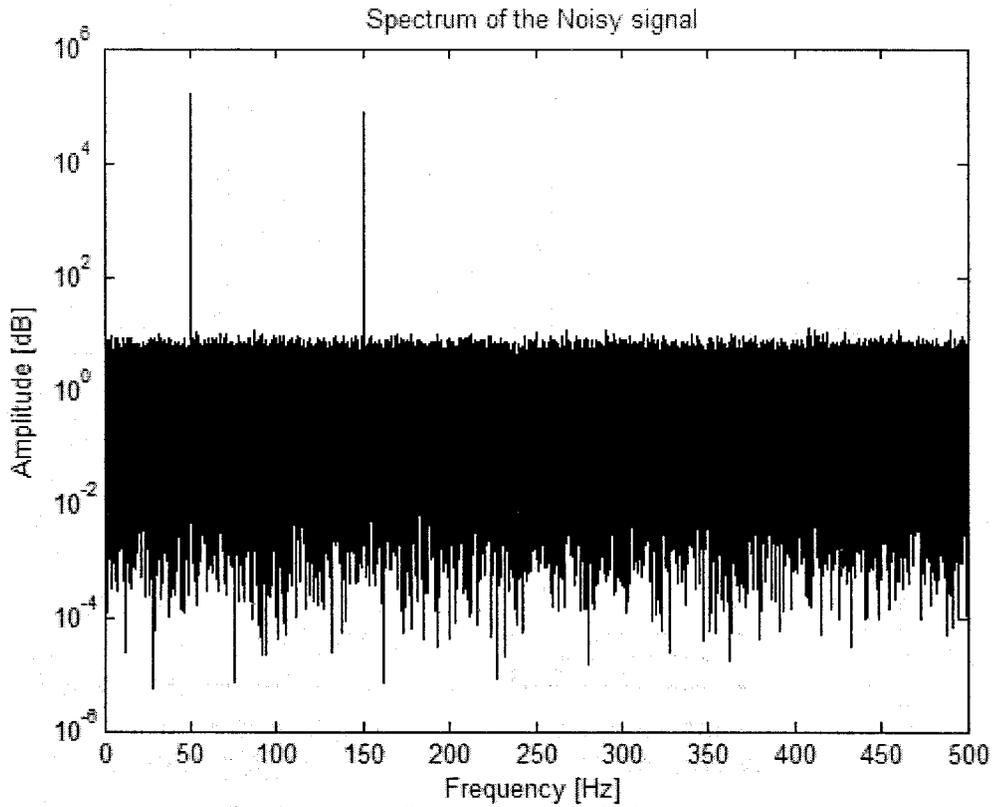
Figure 2. Power spectral density of the simulated data.

As seen from the Figure 2, two sinusoidal components, which are given at 50 Hz and 150 Hz, are fundamental frequency and its third harmonic respectively.

Here, to determine the effect of the noise term if we compute the statistical parameters of the simulated data as defined in section 3. These computed parameters can be given by table 1.

Table 1. Computed statistical parameters.

Statistical Parameters	Mean (μ)	Standard Deviation (σ)	Skewness (s)	Kurtosis (k)
	0.0018	1.3201	2.0040e-004	2.7303

According to result of these parameters the signal characteristic is almost in the Gaussian form. This situation is related to the noise gain. If the signal to noise ratio (SNR) is computed, then it can be given by the following formulation [5, 6].

$$SNR \ [dB] = 10 \ Log_{10} \ (Power \ of \ Signal \ / \ Power \ of \ Noise)$$

Where the signal or Noise power can be indicated by the variance of the signals for this reason if it is computed, the SNR can be given by -1.2741. This result is a small number and it denotes that the Gaussian noise is dominant in the simulated data. Also this property can be shown by the variation of the 1/f-spectrum.
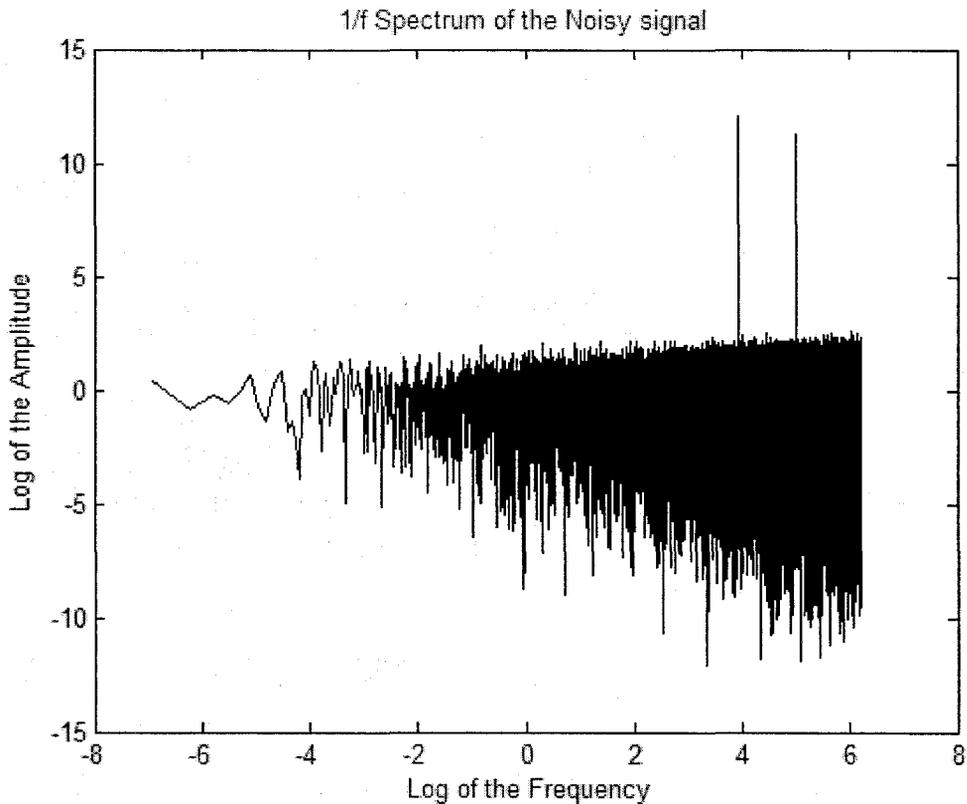

Figure 3. 1/f spectrum of the simulated signal.

In figure 3, the high frequency region indicates the Gaussian noise as well as sinusoidal components at 50 and 150 Hz. Also, the probability density function of the simulated data, which is defined by the statistical parameters computed in Table 1, is shown by the following figure.
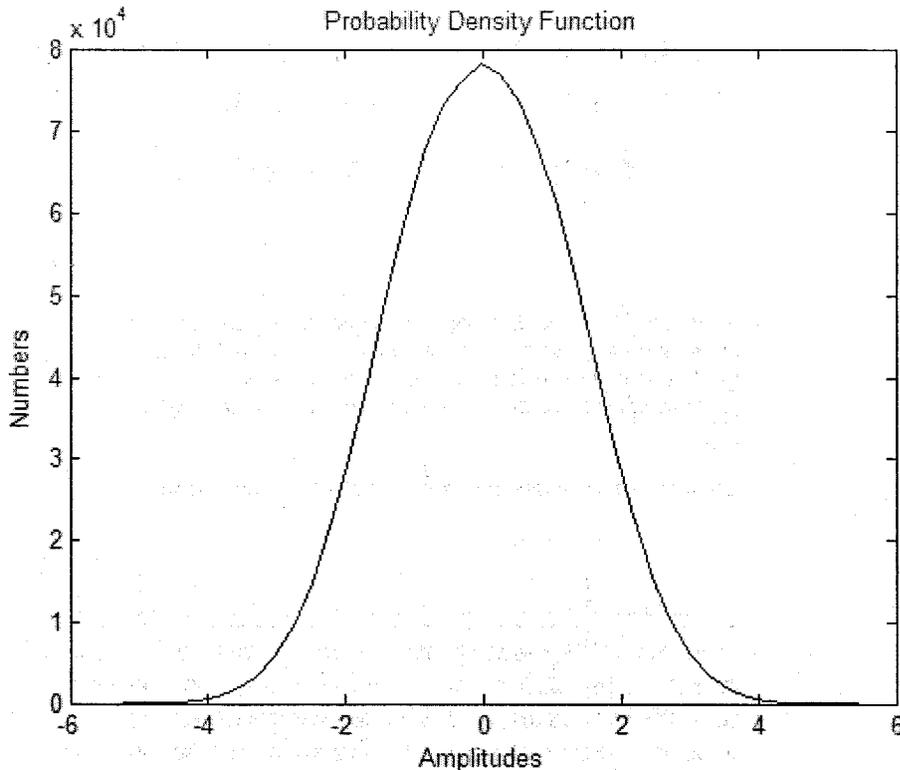
Figure 4. The probability density function of the simulated signal.

6. CONCLUSIONS

This study is a fundamental research to show the noisy signal characteristics related to the spectral and statistical properties. Under the assumption of the existence of signals that reflect the basic properties of special physical processes, this signal analysis application was realized for both of the spectral and statistical techniques. In this sense, most interesting view-point of this study is 1/f spectrum because it combines the properties of the spectral and statistical quantities. At the same time it shows the flat noise spectrum, which denotes the white noise. Also, in the probabilistic manner, Figure 4 shows the bell-shaped variation of the analyzed signal as a highly symmetrical distribution function

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