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Some Results of the Caratheodory's Class

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Abstract: In this paper we investigate some results on the Caratheodory's class. Let $f(z)=1+p_1z+p_2z^2+p_3z^3+\ldots$ be analytic in the unit disc D= $\{z/|z|\leq 1\}$. and satisfying the conditions $f(0)=0, \operatorname{Re} f(z)\geq 0$ .then the function $f(z)$ is called the Caratheodory function.

Özet: Bu çalışmada Caratheodory sınıfı üzerine bazı sonuçları araştırırız. $f(z)=1+p_1z+p_2z^2+p_3z^3+\ldots$ fonksiyonu birim dairede analitik olsun ve $f(0)=1, \operatorname{Re} f(z)\geq 0$ koşullarını gerçeklesin bu durumda $f(z)$ fonksiyonu Caratheodory fonksiyonu olarak adlandırılır.

Key words and phrases: Positive real part.Caratheodory class, Coefficient inequality

Introduction

Let $f(z)=1+p_1z+p_2z^2+p_3z^3+\ldots$ be analytic in the unit disc D and satisfying the conditions $f(0)=1, \operatorname{Re} f(z)\geq 0$ then the function $f(z)$ is called Caratheodory function .The class of these functions is denoted by P.

Primary Results

In this section of this paper we shall give some lesmmas for the class P.

Lemma I. $f_b(z)=\dfrac{1-z^2}{1-bz+z^2}\in$ P.

Proof I. if we take $z=1.e^{i\theta}$ we obtain that

$$p_b(e^{i\theta})=\frac{1-(e^{i\theta})^2}{1-b.e^{i\theta}+(e^{i\theta})^2}=\frac{1-e^{2i\theta}}{1-b.e^{i\theta}+e^{2i\theta}}=\frac{1-(Cos2\theta+i.Sin2\theta}{1-b(Cos\theta+i.Sin\theta)+(Cos2\theta+iSin2\theta)}=$$

$$\frac{(1-Cos2\theta)-i.Sin2\theta}{(1-bCos\theta+Cos2\theta)+i(Sin2\theta-b.Sin\theta)}=\frac{4.Sin^2\theta.Cos^2\theta-2.bSin^2\theta.Coc\theta-4.Sin^2\theta.Cos^2\theta+2.bSin^{2\theta}Cos\theta}{(2.Cos\theta-b)(Cos^{2\theta}+Sin^{2\theta})}=0$$

Since the minumum of a harmonic function occurs on the boundary, we have

$$\operatorname{Re}\left(\frac{1-z^2}{1-b.z+z^2}\right)\geq 0$$

for $|z|\leq 1$.This shows that the lemma is true.

Lemma Iı. The function

$$f_0(z)=\frac{1-z^2}{z}-\frac{(1-z)^2}{z}.\frac{1-z^2}{1-b.z+z^2}$$

satisfies $\operatorname{Re} f_0(z)=0$, for $|z|=1$.

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Proof: $f_0(z)=\frac{1-z^2}{z}-\frac{(1-z)^2}{z}.\frac{1-z^2}{1-b.z+z^2}=\frac{(2-b)(1-z^2)}{1-b.z+z^2}$

Then we have for $|z|=1$, $f_0(|z|=1)=0$

This shows that for the boundary value of the unit disk $\operatorname{Re} f_0(z)=0$.

Corollary I . The function $f(z)=\frac{1-z^2}{z}-\frac{(1-z)^2}{z}.p(z), p(z)\in P$ satisfies $\operatorname{Re} f(z)\geq 0$, $f(0)=1$, and $f(z)$ is analytic in the unit disk. Therefore from the definition Caratheodory class, and the definition of the harmonic functions we have $f(z)\in P$ .This corollary was proved by M.S.Robertson [3]

Definition I. Now we define the class of functios which are analytic in the unit disk D and satisfies the condition $f(0)=1, \operatorname{Re} f(z)\geq 0$, and

$$f(z)=1+bz+\sum_{n=2}^{\infty}p_n z^n$$

This class is a subclass of Caratheodory class. Where b is fixed coefficient.This class is denoted by P_B.

Corollary II. From the lemma I, lemma II, corollary I, and definition I, then we have If $f(z)\in P_B$ then the function

$$f_1(z)=\frac{1-z^2}{z}-\frac{(1-z)^2}{z}.f(z)$$

has the Taylor expansion

$$f_1(z)=\frac{1-z^2}{z}-\frac{(1-z)^2}{z}(1+bz+\sum_{n=2}^{\infty}p_n z^n)\Rightarrow f_1(z)=(2-b)+\sum_{n=1}^{\infty}q_n z^n$$

therefore, the function $f_2(z)=\frac{f_1(z)}{2-b}=1+\sum q_{*n}z^n$

is analytic in the unit disk D and satisfies the conditions $f_2(0)=0, \operatorname{Re} f_2(z)\geq 0$

Therefore, if we use the definition of the cararheodory functions, we obtain that $f_2(z)\in P$.

Corollary III. From the corollary II, then we have $|q_{*n}|\leq 2.|2-b|$.

This is the generalized Caratheodory coefficient inequality.

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