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FRACTAL PROPERTIES OF EARTHQUAKES IN MARMARA

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Abstract
Many geophysical phenomena such as frequency-size distribution of earthquakes, faults, and fractures show multifractal properties. Having fractal distribution requires that number of objects larger than a specified size has power-law dependence. Using power-law relations for statistics of seismicity is not a new subject in geophysics. The Gutenberg and Richter's b-value, which is a good example, defines a relationship between occurrence rates and magnitudes of earthquakes and has been used as the indicator of seismic activity. This study aims to better understand the occurrence systems of earthquakes around Marmara Sea and the seismic behaviour of the investigation region, by means of multifractals and b-value.

Özet
Depremlerin büyüklük-frekans dağılımı, faylar ve kırıklar gibi birçok jeofiziksel olgu multifraktal özellikler gösterirler. Fraktal dağılım, belirli bir büyüklükten daha büyük nesnelerin sayısının, büyüklükleri ile üstel ilişki göstermesini gerektirir. Deprem etkinliğinin istatistiksel olarak incelenmesinde üstel ilişkilerin kullanılması, jeofizikte yeni bir konu değildir. Deprem oluşum sayıları ve büyüklükleri arasındaki ilişkiyi tanımlayan Gutenberg ve Richter'in b-değeri buna güzel bir örnektir ve sismik etkinliğin göstergesi olarak kabul edilir. Çalışma, Marmara Denizi civarında oluşmuş depremlerin oluşum sistemlerinin ve inceleme alanının sismik davranışının, multifraktal yapılar ve b-değeri yardımıyla daha iyi anlaşılmasını amaçlamaktadır.

Key words: Earthquake, multifractal analysis, fractal dimensions, b-value, Marmara.

1. Introduction

The phenomena such as earthquakes and faults are "self similar" systems at any scale and can be described by a power-law relation. The exponent of this relation is called as the fractal dimension and it shows the "degree of complexity" in a chaotic system. In general terms, fractals are described as in Equation 1, where N is the number of objects (boxes) with a size greater than the given size of r, C is the ratio constant, and D is the fractal dimension [7, 9].

$$N = \frac{C}{r^D} \tag{1}$$

For multifractal systems, higher order moments of Equation 1 are used, in order to analyse the behaviour of objects which form the system [9]. The capacity dimension (D_0) is the same as the general fractal dimension mentioned in Equation 1. It does not include frequency information. The information dimension (D_1) is a special form of the capacity dimension and is used as a measure of "expectation". The correlation dimension (D_2) is the last fractal dimension type used in this study and unlike the capacity dimension, it contains frequency information [8, 9].

It is possible to obtain all three dimensions using Equation 2 [2, 6]. For D_1, lim→1 form of the equation is used. In the Equation 2, C (r) is the correlation integral, r is the "box" size, and q is the order of the moment. The correlation integral is given in the Equation 3 where N is the number of objects and H(x) is the Heaviside function.

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$$D_q = \frac{\log C_q(r)}{\log r} \tag{2}$$

$$C_q(r) = \left\{ \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{N}\sum_{j=1}^{N} H\left(r - \left|x_i - x_j\right|\right)\right]^{q-1} \right\}^{1/(q-1)} \tag{3}$$

The functionality of Heaviside function is to count the data points i.e. earthquake epicenters. The difference of coordinates, which is used in Heaviside function, is obtained by Equation 4, where ϕ and θ represent latitude and longitude of earthquake epicenter respectively, in spherical coordinate system.

$$x_i - x_j = \cos^{-1}\left(\cos\varphi_i.\cos\varphi_j + \sin\varphi_i.\sin\varphi_j.\cos\left(\theta_i - \theta_j\right)\right) \tag{4}$$

Although the term of fractals was introduced to the world by Mandelbrot in 1967 [5], power-law relations in order to investigate the geophysical phenomena were used earlier. Gutenberg and Richter [3] defined a relationship between the frequencies and magnitudes of earthquakes (Equation 5) in 1954. The slope of the plot of occurrence numbers against magnitudes is known as the b-value and has been used as the indicator of seismic activity of studied region.

$$\log N = -bm + \log a \tag{5}$$

In the Equation 5, N is the number of earthquakes which have magnitudes greater than a given m magnitude for unit time, and a and b are constant coefficients. The relationship between the generalized fractal dimension and b-value is approximately found as D = 2b by Aki [1], by using worldwide earthquakes [9].

In the present work, for investigating the possibility of earthquake prediction and constructing an infrastructure of the objective, the spatial distribution of earthquakes around Marmara Sea is analysed using the self similarity property of fractals. The main objective is gathering the parameters to develop a deterministic fractal model which will be suitable for defining the earthquake occurrence in the region.

2. Data and calculation

The earthquake data is obtained from the web site of Kandilli Observatory and Earthquake Research Institute [4]. The data for the period between 1975 and 2005 is used in calculations. The spatial distribution of earthquakes with magnitudes (M) greater than or equal to 2.0 is shown in Figure 1.

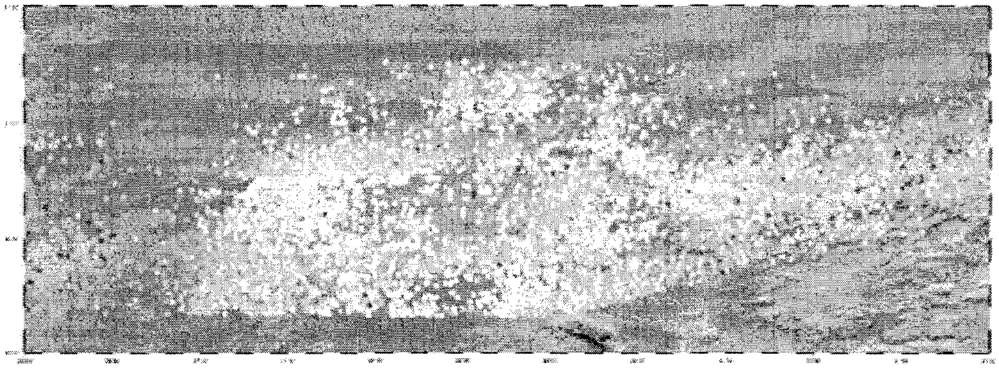

Figure 1. The epicentral distribution of earthquakes from 1975 to 2005 (M $\geq$2.0). The earthquakes with the magnitudes of 2.0 $\leq$M < 2.7, 2.8 $\leq$M < 4.0 and M $\geq$4.0 are shown with white, light gray and dark gray dots, respectively. The figure is prepared by public domain GMT software [10].

Since this is a preliminary study which aims to investigate the possibility of producing a deterministic fractal model for earthquake occurrence, earthquake data is limited with the year of 2005. The latter data is planned to be used for the comparison purposes of theoretically produced data from the deduced model. The occurrence time of the first earthquake in 1975 (02 Jan. 1975, 00:21:57.1 local time) is accepted as the starting time and the "Box Counting Method" is used in fractal dimension calculations.

The real-time changes of D_0, D_1 and D_2 obtained by the exact earthquake occurrence times are given in Figure 2. There are 469 earthquakes with zero magnitude in the catalogue, which cannot be used in b-value calculations. Therefore, the cutoff magnitude is obtained as 2.0 by the frequencies of earthquakes with magnitude (Figure 3), in order to eliminate these erroneous records. In addition, in order to be independent from the time, the calculations should be performed per a certain time increment, which is accepted as 0.5 days in the study. Thus, the fractal dimensions are re-calculated including the b-value, with the time interval and cutoff magnitude (Figure 4).

As no certain scaling signatures are found for the earthquake epicenters (Figure 4), the calculations are renewed for the depth intervals which are determined by considering the event frequencies with depth (Figure 5) as 0-4 km, 4.1-9 km, 9.1-16 km and greater than 16 km. All calculations are performed again for these intervals and results are shown in the following.

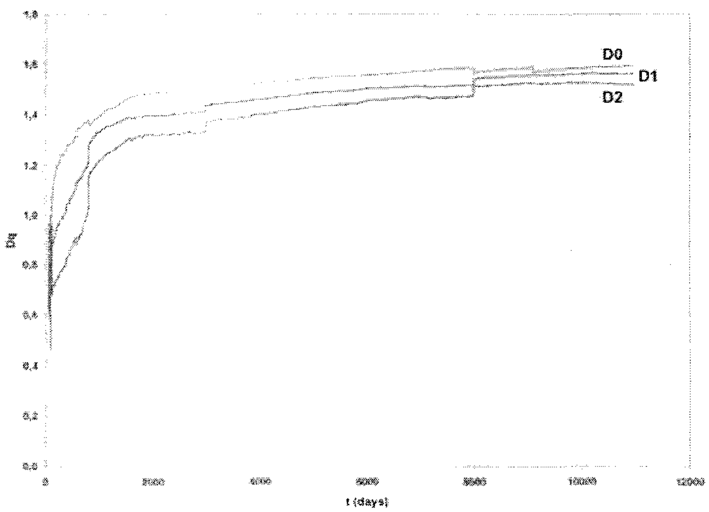

Figure 2. The real-time fractal dimensions calculated by exact earthquake occurrence times.

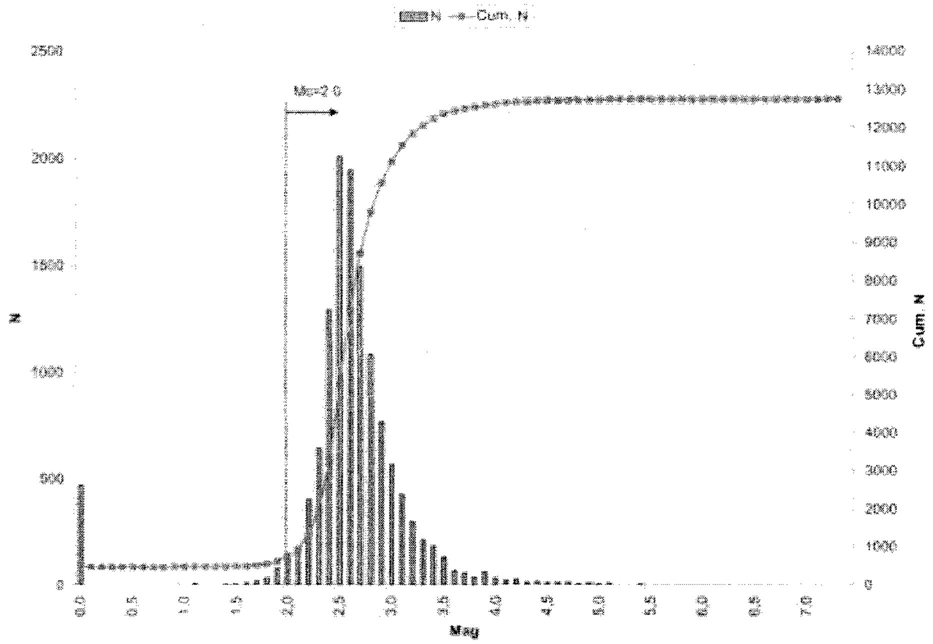

Figure 3. Number of earthquakes against magnitude. The dashed line with a right arrow marks the cutoff magnitude of $M_c = 2.0$. The continuous line with dots indicates the cumulative number of earthquakes.

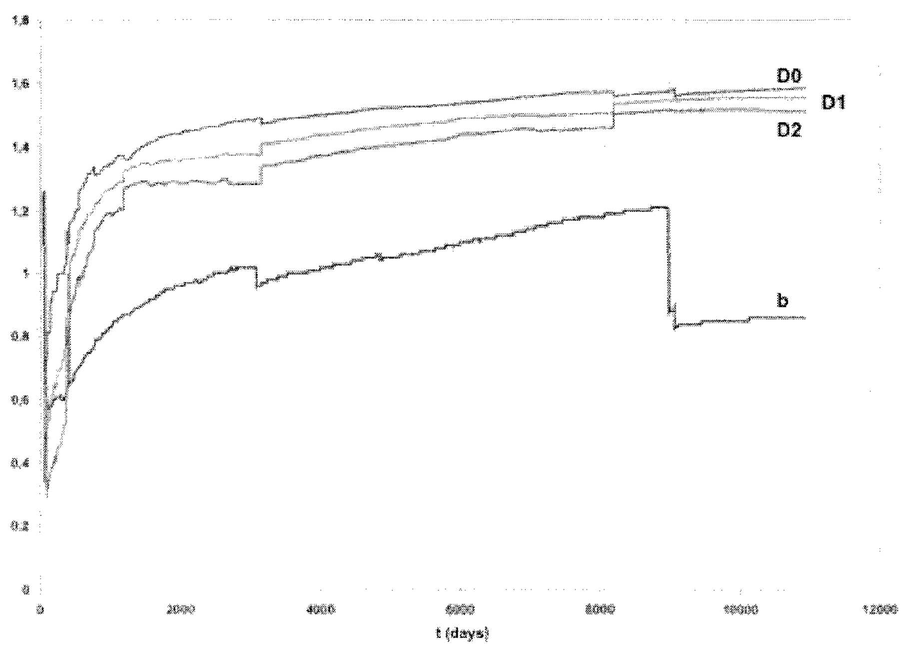

Figure 4. The fractal dimensions and b-value results with cutoff magnitude of 2.0 and time interval of 0.5 days.

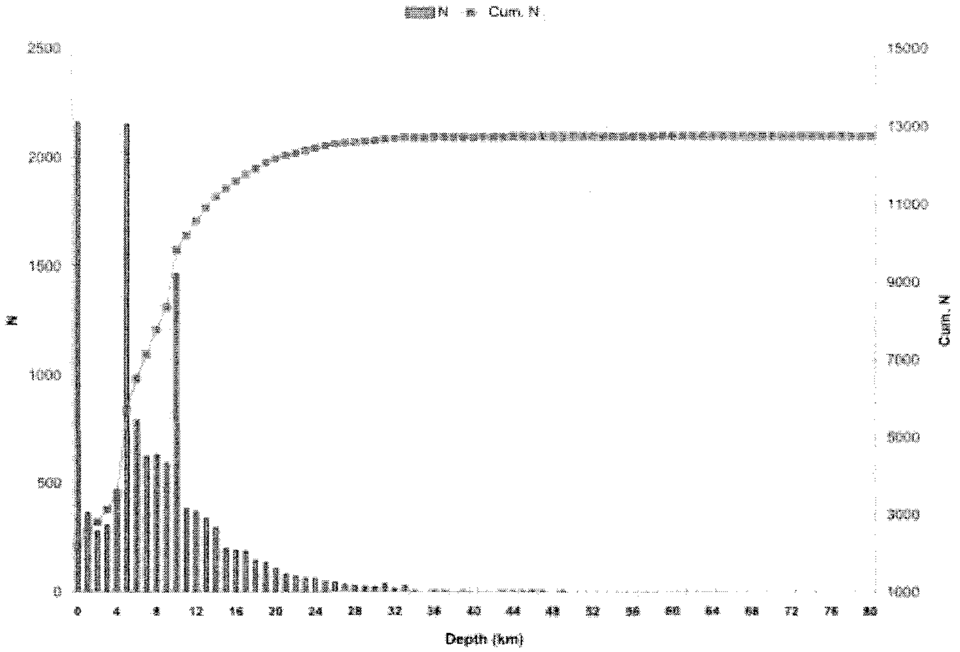

Figure 5. Number of earthquakes against depth values. Continuous line with dots indicates the cumulative number of earthquakes.

The comparison of Figure 2 and Figure 4 shows that choosing the cutoff magnitude as 2.0 and the time interval as 0.5 days is a good approximation. The graphs are almost identical except nearly the first 2000 days as expected, since the earthquakes with zero magnitudes are mostly in this period. Both graphs show similar patterns, but there is a time shifting for the anomalies at around 8000 days in Figure 4.

Two dimensional fractal analyses is preferred instead of three dimensional to understand the behaviour of earthquake distribution at certain depth levels, rather than in a volume. All calculations are performed by using the data which starts from the beginning of the investigation time period. For each time interval of calculation, new earthquakes are added to previous data, in order to trace how scaling properties of epicentral distribution and b-value change by the newly added data.

In general, the coordinates of active faults are used to obtain the capacity dimension as the capacity elements in earthquake phenomena are faults. In the present work, the earthquake epicenters are used instead of fault coordinates, in order to avoid possible errors while digitizing and inexactness of fault knowledge. Besides, earthquakes already occur on faults and there are several geometric models for faulting in the region.

3. Results and conclusion

As the result of two dimensional fractal distribution without considering the depths, D_0, D_1, and D_2 are found to be 1.59, 1.56 and 1.51 for the whole system (Figure 4). Since the dimension values are close to each other, system can be considered as homogeneous and predictable. However, there is no linear relation between b-value and fractal dimensions.

It is observed from the deduced values in respect to depths that there is no linear relation between b-value and fractal dimensions either, but earthquakes at each depth level show multifractal properties (Figure 6, 7, 8, and 9). The possible relations between the sudden changes and earthquake occurrence times are investigated. At all four depth intervals, there is no seismic activity at least for 0.5 days; but it is not possible to maintain a relationship between the fractal dimensions and time.

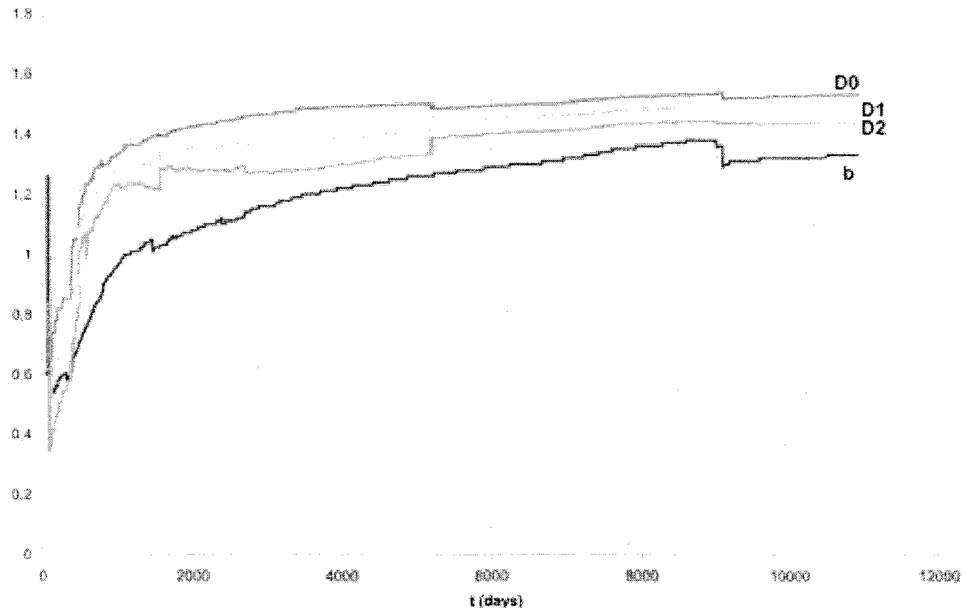

Figure 6. The b-value and fractal dimensions against time (days) for the depth level of 0-4 km.

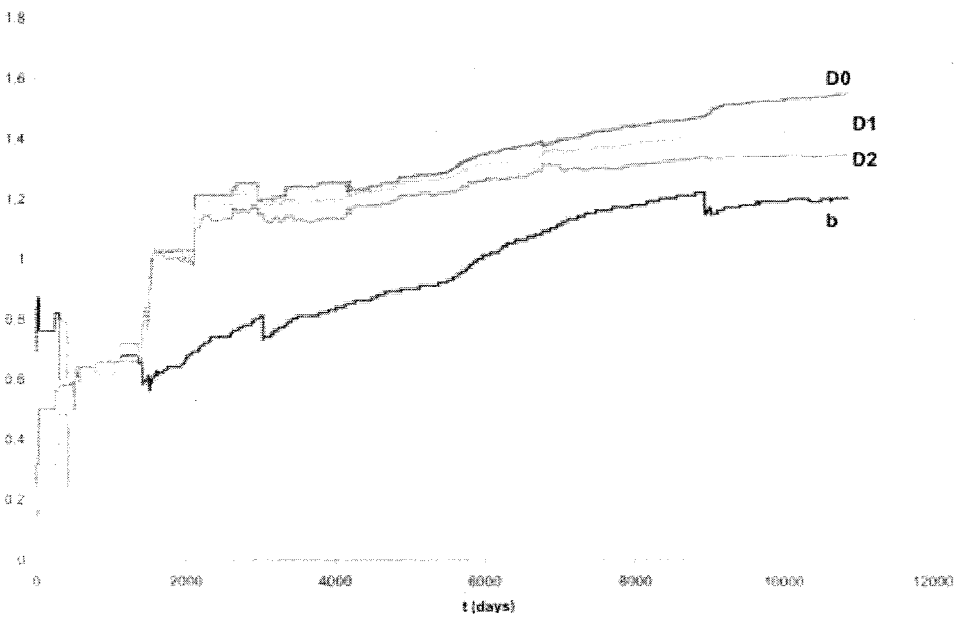

Figure 7. The b-value and fractal dimensions against time (days) for the depth level of 4.1-9 km.

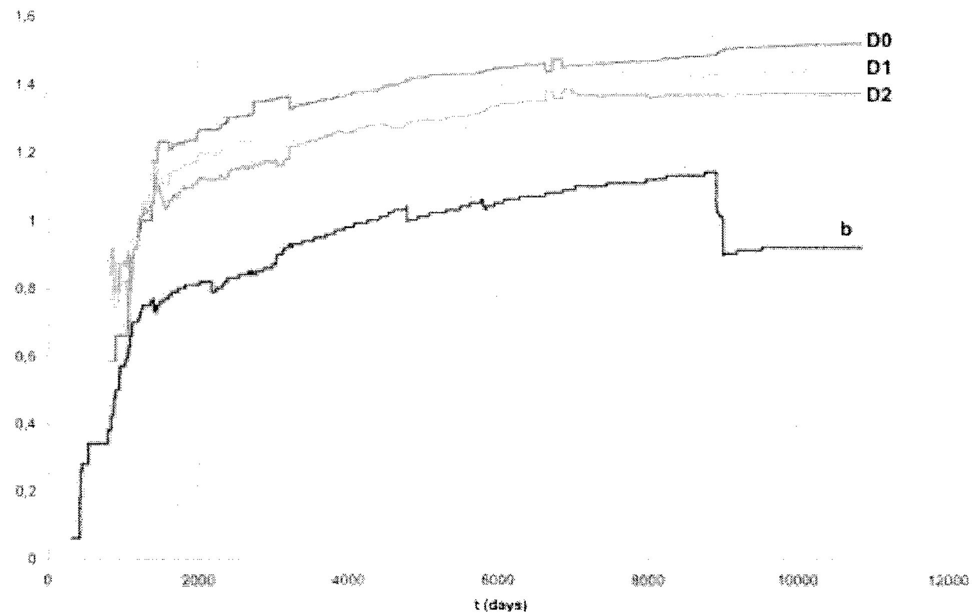

Figure 8. The b-value and fractal dimensions against time (days) for the depth level of 9.1-16 km.

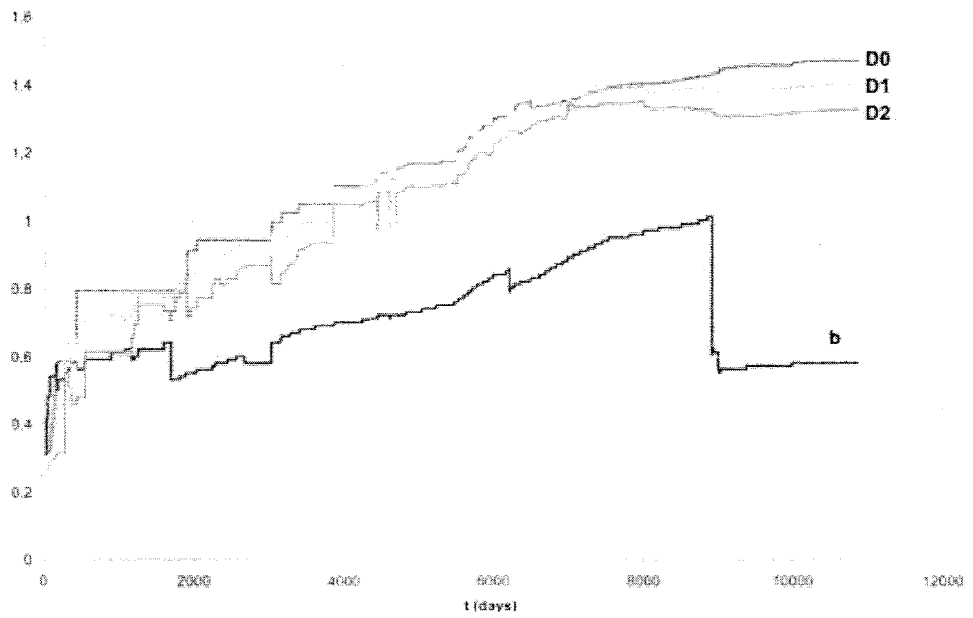

Figure 9. The b-value and fractal dimensions against time (days) for the depth level greater than 16 km.

Especially for the earthquakes at depths greater than 16 km, the fractal dimension values are very close to each other until about 7000 days (Figure 8). However, the decrease in D_1 and D_2 values is interesting after that. This characteristic lasts throughout 2500 days and with the 17 August 1999 Golcuk (M_w=7.4) and 12 October 1999 Duzce (M_w=7.2) earthquakes. This could be an indicator that the variations in the dimension values might be useful for foreseeing the behaviour of the system. These earthquakes cause sharp falls at the b-value graphs obtained for all depth levels, which can be related with high energy release.

When earthquake distribution by the time is monitored, it is seen that earthquake clusters in the eastern and middle parts of the investigation region affect the scaling properties of the whole system in different ways. It could be thought that the characteristics of the faults would produce this effect. Therefore, the geometrical properties of the faults such as strike, dip, and slip, should also be taken into consideration, in order to understand the behaviour of the system better.

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