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Discrete Ordinates Method and the Examination of Some Benchmark Problems

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Abstract
In this study a wellknown numerical solution method of neutron transport equation, Discrete Ordinates Method, has been introduced and using some Benchmark problems the ray-effect distortions observed in the method have been examined. For this purpose, TWOTRAN-II code has been chosen and executed for sample problems. Finally, the effectiveness of the method has been discussed in detail.

Özet
Bu çalışmada, nötron transport denkleminin çok tanınan bir çözüm yöntemi, Kesikli Ordinatlar Yöntemi tanıtılmış ve Benchmark problemleri kullanarak metodda gözlenen ışın-etkisi bozulmaları incelenmiştir. Bu amaçla TWOTRAN-II kodu seçilmiş ve bazı örnek problemler için koşulmuştur. Sonuçta, yöntemin etkinliği ayrıntılı bir biçimde tartışılmıştır.

Keywords: *Discrete ordinates method, neutron transport equation, deterministic numerical method, benchmark problem, ray-effect*

1. Introduction

In nuclear reactor design and analysis, the solution of the neutron transport equation (Eq. 1.1. and 1.2) which determines neutron distribution in a system; such as a nuclear core, plays an important role. However, it is very difficult to solve this equation except for simple model problems, since it has an integro-differential form with seven variables. Other factors have also an effect complicating the solution, such as complex energy dependence of cross sections and geometrical arrangement of the materials in the core, etc. Therefore, the neutron transport equation, together with the boundary conditions (and initial condition, if required), is solved usually by deterministic transport codes [1, 2].

$$\vec{L}\,\phi\,(\vec{r}, E, \vec{\Omega}, t) = Q\,(\vec{r}, E, \vec{\Omega}, t) \tag{1.1}$$

$$\vec{L} = \frac{1}{v}\frac{\partial}{\partial t}\;\vec{\Omega}.\Delta\; + \Sigma_t\,(\vec{r}, E) - \int_{4\pi} d\Omega' \int_0^{\infty} dE' \Sigma_s\,(\vec{r}, E' \rightarrow E, \vec{\Omega}' \rightarrow \vec{\Omega})$$

$$-\,\chi(E) \int_{4\pi} d\vec{\Omega}' \int_0^{\infty} dE' \nu(E')\, \Sigma_f\,(\vec{r}, E') \tag{1.2}$$

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2. The Method

The Discrete Ordinates Method, is one of the wellknown deterministic solution methods of neutron transport equation. The method is straightforward and widely used in transport calculations.

In this method, a set of discrete directions (or ordinates) for $\vec{\Omega}$ is chosen and directional fluxes are evaluated for these directions. The derivatives and integrals appearing in the transport equation must be replaced by a corresponding discrete representation by using finite difference techniques and numerical integration schemes (i.e. standard Gauss quadratures [3]).

In the study, the angular flux is expanded in a series of Legendre polynomials,

$$\phi = \sum_{n=o}^{\infty} (2n + 1) \sum_{k=o}^{n} R_n^k \phi_n^k \tag{2.1}$$

and the expansion coefficients are given by,

$$\phi_n^k = \int_{-1}^{+1} d\mu \int_{0}^{\pi} d\varphi \; R_n^k \phi / 2\pi \tag{2.2}$$

This integral, according to the Gauss-Legendre quadrature (Eq.2.8) consideration is approximated by the sum,

$$N_{nij}^{k} = \sum_{m=1}^{MT} w_m R_n^k \; (\mu_m, \varphi_m) \, N_{ijm} \tag{2.3}$$

where φ_m is defined in the program, TWOTRAN-II [4], by

$$\varphi_m = \tan^{-1} (1 - \mu_m^2 - \eta_m^2)^{1/2} / \eta_m \quad , \quad \eta_m > 0 \tag{2.4}$$

$$\varphi_m = \tan^{-1} (1 - \mu_m^2 - \eta_m^2)^{1/2} / \eta_m + \pi \quad , \quad \eta_m > 0 \tag{2.5}$$

and, in this study there are IT intervals in the x (or r) direction, JT intervals in the y (or z or Θ) direction, and MT solid angle intervals such that

$$\begin{aligned} x_{i-1/2} < x_i < x_{i+/2} \quad &, \; i = 1,2,.........,IT \\ y_{j-1/2} < y_j < y_{j+1/2} \quad &, \; j = 1,2,.........,JT \\ \Omega_{m-1/2} < \Omega_m < \Omega_{m+1/2} \quad &, \; m = 1,2,.........,MT \end{aligned} \tag{2.6}$$

In the program, total source is also given as S_{gijm} by the sum of

$$(SS)_{gijm} = (\text{scatter source})_{gijm} = \sum_{h=1}^{IGM} \sum_{n=0}^{ISCT} (2n+1)\sigma_{snh\to g} \sum_{k=0}^{n} R^k_{nm} N^k_{nhij} \quad (2.7)$$

$$(FS)_{gijm} = (\text{Fission source})_{gijm} = \chi_g \sum_{h=1}^{IGM} \nu\sigma_{fh} N^0_{0hij}$$

$$(IS)_{gijm} = (\text{Inhomogeneous source})_{gijm} = \sum_{n=o}^{IQAN} (2n+1) \sum_{k=o}^{n} R^k_{nm} Q^k_{ngij}$$

$$\int_{-1}^{+1} F(x)\,dx = \sum_{i=0}^{n} w_i F(x_i) \quad (2.8)$$

Although the method has some advantages to solve the transport equation, sometimes some deleterious numerical problems has been met; such as ray-effect. The ray-effect distortions are anomalous ripples that appear in the scalar flux for problems with isolated sources and low scattering rations in two or three dimensional geometries. Several remedies for eliminating or mitigating the ray-effect problem have been proposed. It seems that the practical remedy for the ray-effect is to increase the number of directions. In general, when this is done, the oscillation frequencies become higher and they can be confused with physical ones. In recent years, considerable effort has been expended in deriving approaches that eliminate or strongly mitigate the ray-effect distortions [5,6,7].

3. The Results of Benchmark Problems

In this study one of the wellknown two dimensional S_N codes [8-9], TWOTRAN-II has been chosen. A wellknown benchmark problem [7] given in (x-y) cartesian geometry (Problem 1) has also been chosen as a test problem. In the problem, there is a flat isotropic source at the left-bottom corner of a square region. The boundary conditions have been given as reflective on the bottom and the left sides, and vacuum on the top and the right sides of the region (Fig.1). The problem parameters were given in the table.

1.

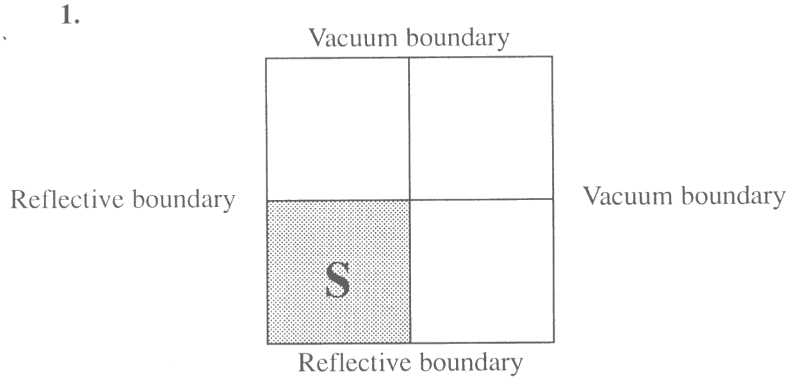

Fig.1. Sample problem in (x-y) geometry (Problem 1)

The code has been executed for this problem and the distribution of scalar flux is examined for different S_N approximations as seen in the Fig.2.

Cross-sections	
Σ_a	0.25
$\nu\Sigma_f$	0.0
Σ_t	0.75
Σ_s	0.50
S	1.0

Table 1. Data for sample Problem 1.

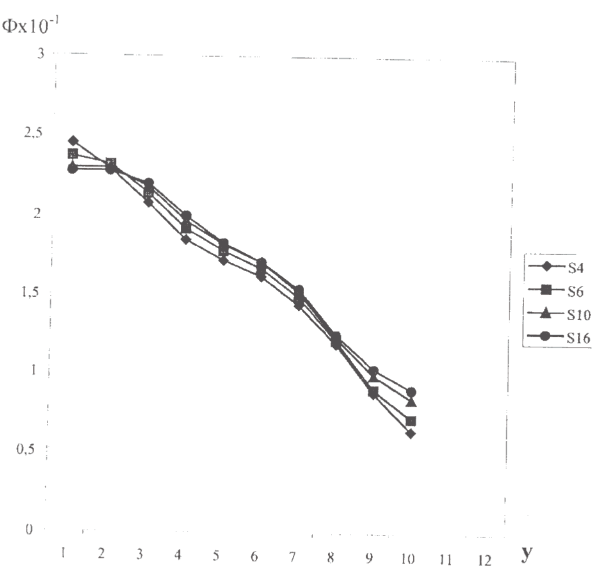

Fig.2 Scalar Flux distribution for (x-y)geometry

Then, problem 1. is adapted to (r-z) cyclindrical geometry, Problem 2 (Fig.3). The values of problem parameters given in the Table 1 were also valid for this problem. This problem has also° been examined in a similar way and the scalar flux distribution for different S_N approximations has been given in the Fig. 4.

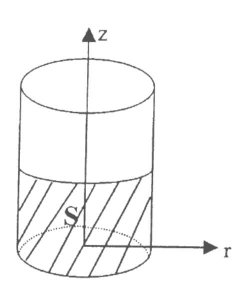

Fig.3. Sample problem in (r-z) geometry (Problem 2)

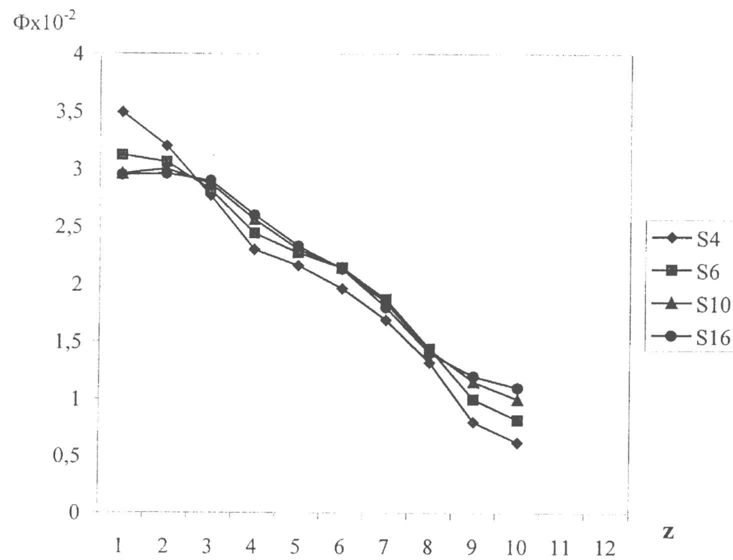

Fig.4. Scalar flux distribution for (r-z) geometry

At the end of the study, another wellknown code, FENT [8] has also been taken into account. Then, all the results obtained by TWOTRAN-II have been compared with the results of FENT (Fig.5).

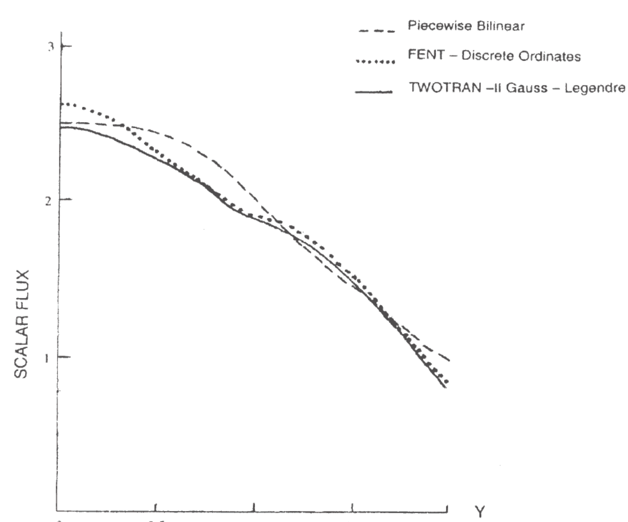

Fig.5. Comparison of the result of TWOTRAN II and FENT codes for (x-y) geometry.

4. Conclusion

At the end of this study, by comparing all the results obtained by two codes for two geometries we concluded that the Gauss-Legendre approximation for angular flux and source, as we used in the study is more effective in cartesian geometry. As seen in Fig.2 and Fig.4 the ray-effect distortions observed on the results of (x-y) geometry are more mitigated by this approximation. Fig.5 which gives some S_4 qraphs of various approximations in the literature is also proved this comment.

Finally, we recommend the Gauss-Legendre quadrature approximation especially for cartesian geometry problems. For other geometries, i.e cyclindrical geometry as given in this study, another approximations can be more useful. As a matter of fact, nowadays in most S_N codes, some hybrid [1,10] techniques for different geometries can frequently be encountered.

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