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COEXISTING ATTRACTORS IN AN AUTONOMOUS 4th ORDER, NON LINEAR, ELECTRIC CIRCUIT

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Abstract:

In this paper we have studied the dynamic behavior of a non driven electric circuit of 4th order with two active elements, one linear negative conductance & one non linear resistor of N-type exhibiting a symmetrical piecewise linear v-i characteristic. The resistor R_1 serves as the control parameter of the system. We have observed formation of "bubbles" for some initial conditions. In a narrow region of R_1 values what has been observed is antimonotonicity in the bifurcation diagram, different routes to chaos via period doubling sequences and reverse period doubling and transition from periodic to quasi-periodic and finally to chaos. We have also studied the dependence of circuit behavior on the initial conditions.

Key-words: *Bubbles, Coexisting attractors, Antimonotonicity, Chaos, Period doubling.*

1. Introduction

The last twenty years has been characterized by the huge development in the study of phenomena with complicated or chaotic behavior. Chaos is a noise like phenomenon that is a product of the inevitable nonlinearity of physical systems. The science of chaos is relatively new and many scientists have observed chaotic behaviors in different areas of science. Firstly physicians and mathematicians mainly dealt theoretically with the study of chaotic phenomena. Theoretical study or simulation of these chaotic phenomena was much easier than study in experimental circuits because of difficulty in searching of chaotic behaviors.

The main advantage of using electric circuits in studying dissipative systems is the easy of implementation using low cost materials. With those elements, we implement a variable of nonlinear resistors and negative conductance. Chaotic phenomena such us bifurcation, period doubling and reverse period doubling, antimonotonicity [1] that has been reported in the literature, have all been observed in electric circuits e.g., the period-doubling route to chaos[2, 3, 6, 8, 11], quasiperiodicity route to chaos [4] . Cascades of period-doubling bifurcations have long been recognized to be one of the common routes to chaos, as exemplified by the one-dimensional logistic map $x_{n+1} = \lambda x_n(1 - x_n)$. As the parameter λ in the logistic map is increased, it is known that periodic orbits are only created but never destroyed. Unlike the monotone bifurcation behavior of the logistic map, it has been showed [7] that in many common nonlinear dynamical systems periodic orbits must be both created and destroyed infinitely often, as the parameter is increased near certain common parameter values. Dawson et al. [7] named this concurrent creation and annihilation of periodic orbits antimonotonicity.

Bier and Bountis [5], demonstrated that reverse period-doubling sequences can be expected to occur, when a minimum number of conditions are fulfilled. Their main result is that a reverse period-doubling sequence is likely to occur in any nonlinear system, where there is a symmetry transformation, under which the state equation remains invariant.

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Indeed our system under the transformation

$v_{C1} \rightarrow -v_{C1}$, $v_{C2} \rightarrow -v_{C2}$, $i_{L1} \rightarrow -i_{L1}$, $i_{L2} \rightarrow -i_{L2}$, $i \rightarrow -i$,

remains invariant.

Chua's circuit family which was created during the effort to make real the Lorentz model is the most classical example route to chaos. In previous studies we have realized many modified Chua's circuits [8-9]. That makes these circuits different is that the changes of values of capacitance, resistance and inductors cause many different dynamic behaviors.

Moreover the behavior of the systems very sensitive to initial conditions (specially the autonomous circuits). The study of Chua's circuit family presents inexhaustible wealth of dynamical behavior and their study has not been completed.

In a recent paper we studied the dynamics of a forth order non autonomous nonlinear electric circuit (driven by a sinusoidal voltage source). In this paper we study the dynamics of a fourth-order autonomous nonlinear electric circuit (Figure 1) with two active elements, one nonlinear resistor R_N with a symmetrical piecewise linear v - i characteristic (Figure 2) and one linear negative conductance (Figure 3).

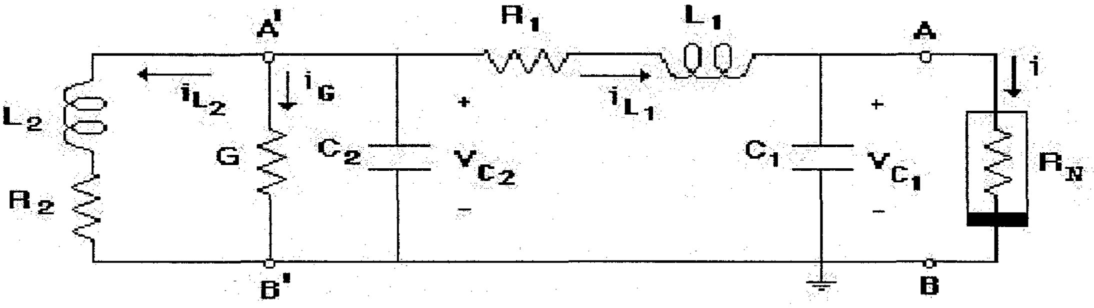


Figure 1. The 4th order autonomous non linear electric circuit

The values of the circuit parameters are:

$R_1 = 0.4 - 1.1K\Omega$, $R_2 = 90\Omega$, $L_1 = 33mH$, $L_2 = 100mH$, $C_1 = 6.6nF$, $C_2 = 15nF$.

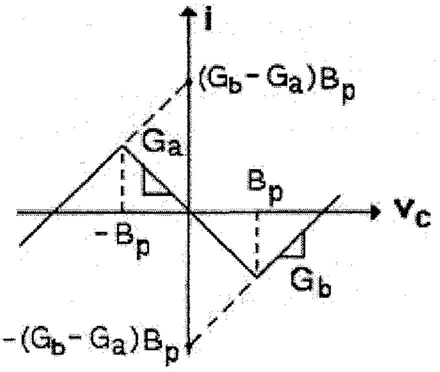


Figure 2. v-i characteristic of non linear resistor R_N

$G_a = -0.105mS$, $G_b = 7mS$, $B_p = 0.68V$

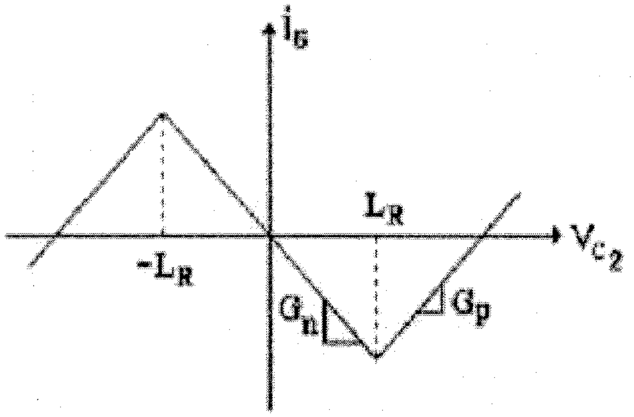


Figure 3. v-i characteristic of negative conductance G
G_n= -0.45mS, G_p= 0.45mS, L_R=7.5V

2. Dynamics of the Circuit

We have chosen the following values for the circuits parameters:
R_2=90Ω, L_1= 33mH, L_2=100mH, C_1= 6.6nF, C_2=15nF.

The values for the non linear resistor R_N are: G_a = -0.105mS, G_b = 7mS, B_p =0.68V and the ones for the negative conductance G are: G_n= -0.45mS, G_p= 0.45mS, L_R=7.5V

The state equations of the circuit are:

$$\frac{du_{C1}}{dt} = \frac{1}{C_1}(i_{L1} - i)$$

$$\frac{du_{C2}}{dt} = -\frac{1}{C_2}(G \cdot u_{C2} + i_{L1} + i_{L2}) \qquad (1)$$

$$\frac{di_{L1}}{dt} = \frac{1}{L_1}(u_{C2} - u_{C1} - R_1 i_{L1})$$

$$\frac{di_{L2}}{dt} = \frac{1}{L_2}(u_{C2} - R_2 i_{L2})$$

where

$$g(v_{C1}) = G_b v_{C1} + 0.5(G_a - G_b)(|v_{C1} + B_p| - |v_{C1} - B_p|)$$

Systems that appear chaotic behavior are very sensitive to changes of the initial conditions. Different initial conditions will probably create totally different dynamic behavior. In our circuit we have studied the dynamics of the system for selected values of R_1 plotting bifurcation diagrams, phase portraits & Lyapunov exponents for different sets of initial conditions:

(i.c.1 /0V, 0V ,0.5mA, -0.5mA) &. (i.c.2 /-0.706V, 0V, -0.08mA, -0.48mA).

The bifurcation diagrams I_{L2} (mA) vs. R_1 for C_1= 6.6nF and initial conditions 1 (i.c.1) is shown in Figure 4 and the same one for initial conditions 2 (i.c.2) is shown in Figure 5. We start the bifurcation diagrams with different initial conditions. We observe that from R1=800Ω to 712Ω the behavior of the system is completely different. From R_1=712Ω to 600Ω the system appears exactly the same behavior without any external influence.

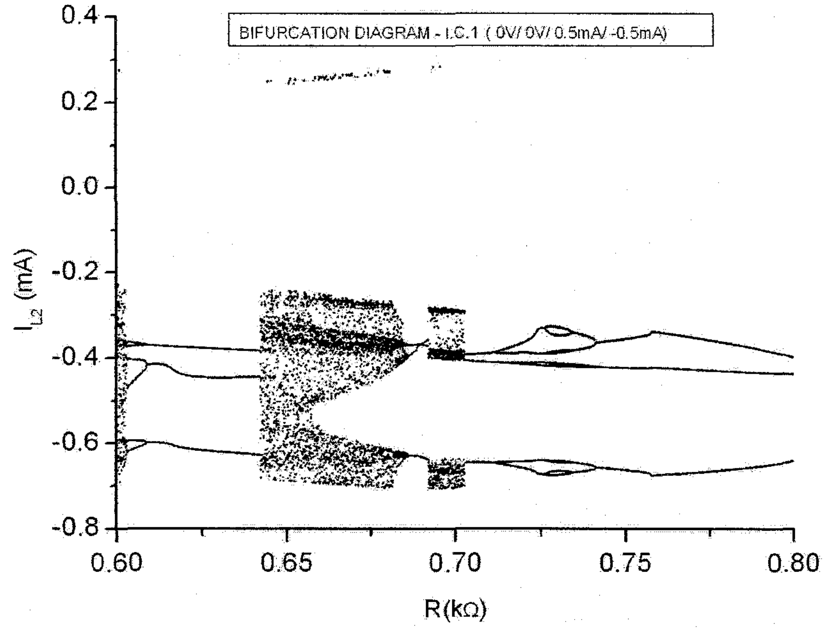


Figure 4. Bifurcation diagrams Ii_{L2} (mA) vs. R_1 for initial conditions 1

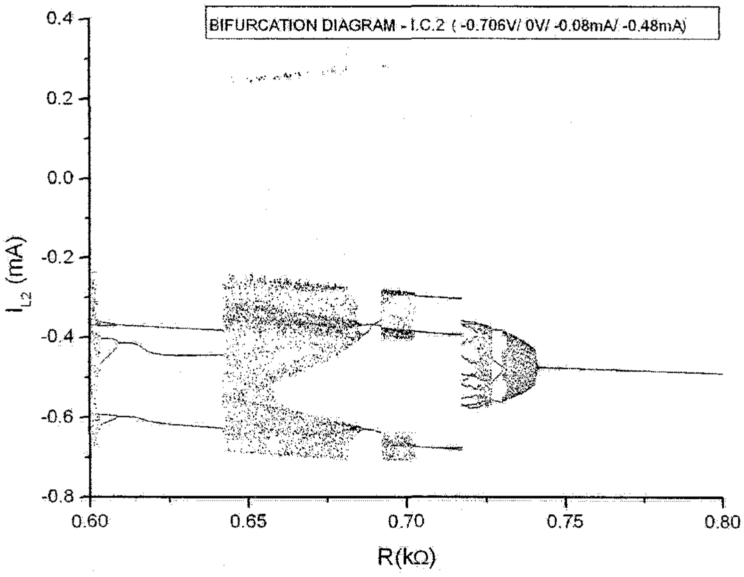


Figure 5. Bifurcation diagrams Ii_{L2} (mA) vs. R_1 for initial conditions 2

Some phase portraits diagrams (V_{C1}-V_{C2}) are shown in Figure 6-9.

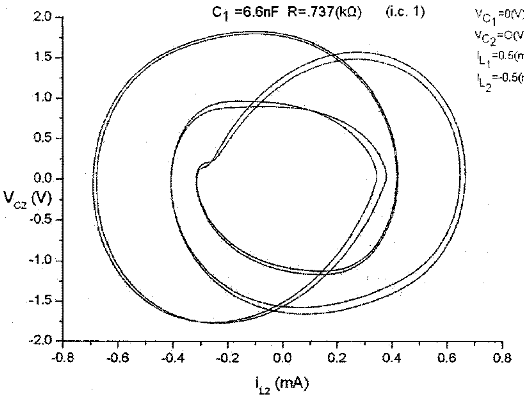


Figure 6

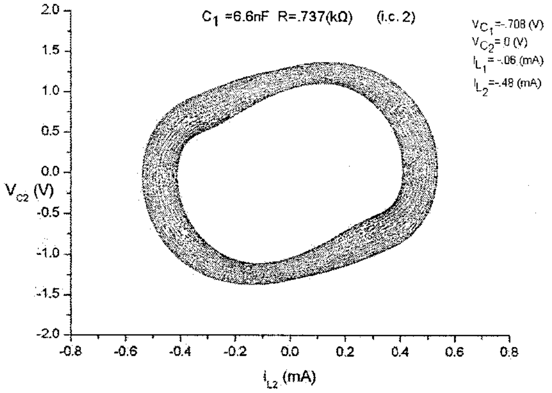
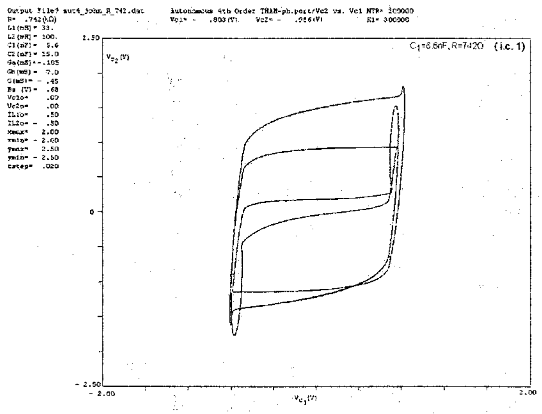


Figure 7



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Output File? aut4_john_R_742.dat      Autonomous 4th Order THAN-ph.port/Vc2 vs. Vc1 NTP= 200000
R=  .742(kΩ)                          Vc1= -   .803(V)    Vc2= -   .956(V)          K1= 300000
L1(mH) = 33.
L2(mH) = 100.
C1(nF) =   6.6
C2(nF) =  15.0
Ga(mS) = -.105
Gb(mS) =   7.0
G(mS) = -  .45
Bp  (V) =   .68
Vc1o=      .00
Vc2o=      .00
IL1o=      .50
IL2o= -    .50
xmax=     2.00
xmin= -   2.00
ymax=     2.50
ymin= -   2.50
tstep=    .020
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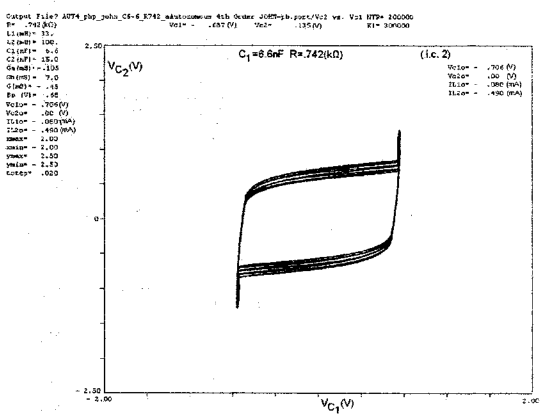
Figure 8



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Output File? AUT4_php_john_C6-6_R742_aAutonomous 4th Order JOHN-ph.port/Vc2 vs. Vc1 NTP= 200000
R=  .742(kΩ)                     Vc1= -   .687(V)     Vc2=    .135(V)          K1= 300000
L1(mH)= 33.
L2(mH)= 100.
C1(nF)=   6.6
C2(nF)=  15.0
Ga(mS)= -.105
Gb(mS)=   7.0
G(mS)= -  .45
Bp (V)=   .68
Vc1o= - .706(V)
Vc2o=   .00 (V)
IL1o= - .080(mA)
IL2o= - .490(mA)
xmax=   2.00
xmin= - 2.00
ymax=   2.50
ymin= - 2.50
tstep=  .020
```

Figure 9

In Figure 6 the phase portrait I_{L2} (mA)–V_{C2} (V) is shown for $R_1 = 0.737K\Omega$ and initial conditions 1 (i.c.1). The state of the circuit is periodic. This is proved by the Lyapunov exponents in Figure 10 (LE1=0, LE2<0, LE3<, LE4<0). In Figure 7 the phase portrait I_{L2} (mA)– V_{C2} (V) is shown for $R_1 = 0.737K\Omega$ and initial conditions 2 (i.c.2). The state of the circuit is quasiperiodic. This is proved by the Lyapunov exponents in Figure 11 (LE1=0, LE2=0, LE3<0, LE4 <0).

Similarly, in Figure 8 the phase portrait I_{L2} (mA)– V_{C2} (V) is shown for $R_1 = 0.742K\Omega$ and initial conditions 1 (i.c.1), the state of the circuit is periodic, and the Lyapunov exponents is shown in Figure 12 (LE1=0, LE2<0, LE3<, LE4<0). In Figure 9 the phase portrait I_{L2} (mA)– V_{C2} (V) is shown for $R_1 = 0.742K\Omega$ and initial conditions 2 (i.c.2).The state of the circuit is quasiperiodic and the Lyapunov exponents is shown in Figure 13 (LE1=0, LE2=0, LE3<0, LE4 <0).

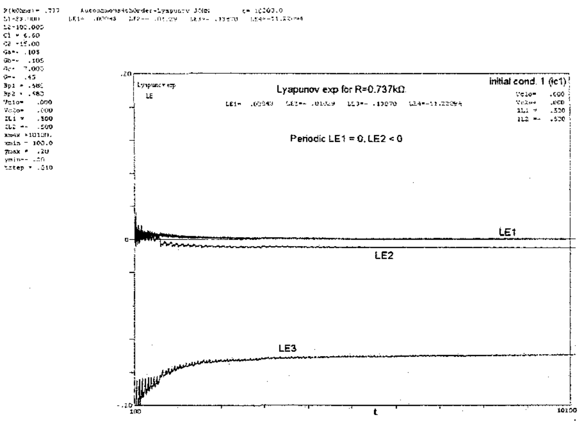

Figure 10

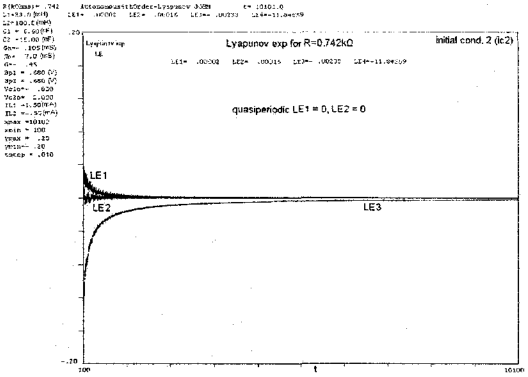

Figure 11

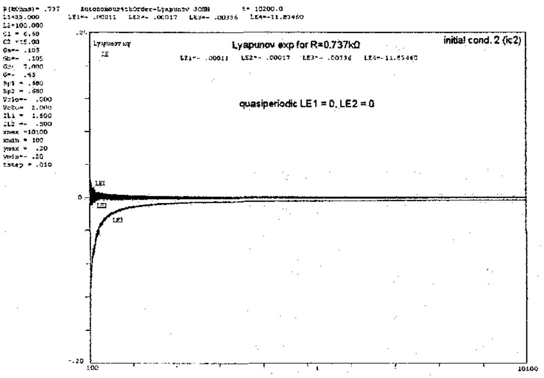

Figure 12

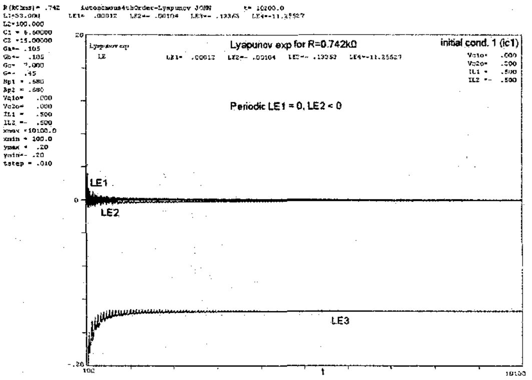

Figure 13

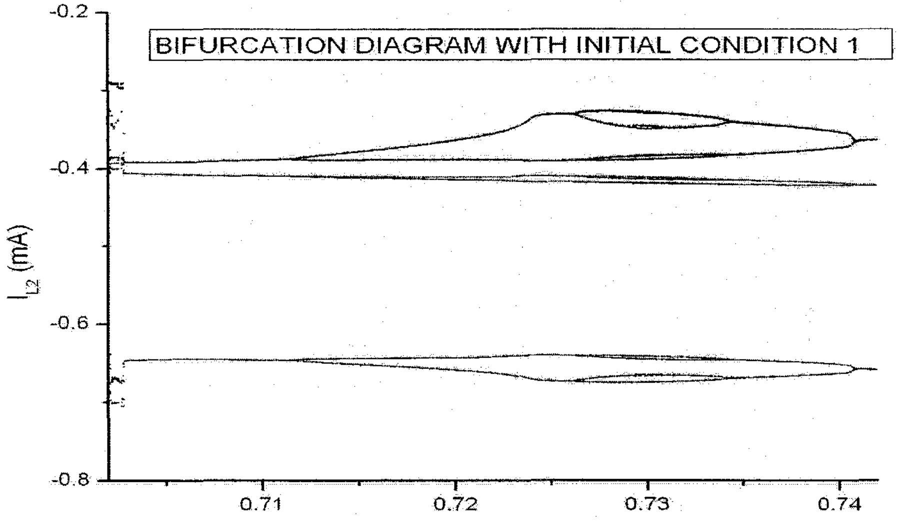

Figure 14

In Figure 14 we have shown the bifurcation diagram for initial conditions 1. As R_1 is increased, the circuit remains in a periodic state according to the following scheme: period-3 → period-6 → period-12 → period-6 → period-3. This scheme is named "triple primary bubble".

- For R= 800 (Ω) – 741.5(Ω), p-3
- R= 741.5(Ω) -734.7(Ω), p-6
- R= 734.7(Ω)-726.6(Ω) p-12,
- R= 726.6(Ω)-712.2(Ω) p-6 and
- R= 712.2(Ω)-703 (Ω), p-3

The triple primary bubble disappears for other initial conditions (like i.c.2). The behavior of the system is completely different. (Figure 4).

In the following figures, we presence the phase portraits for a route to chaos via period doubling in another resistor R_1 range. This route is the same for both initial conditions (i.c.1. & i.c.2.)

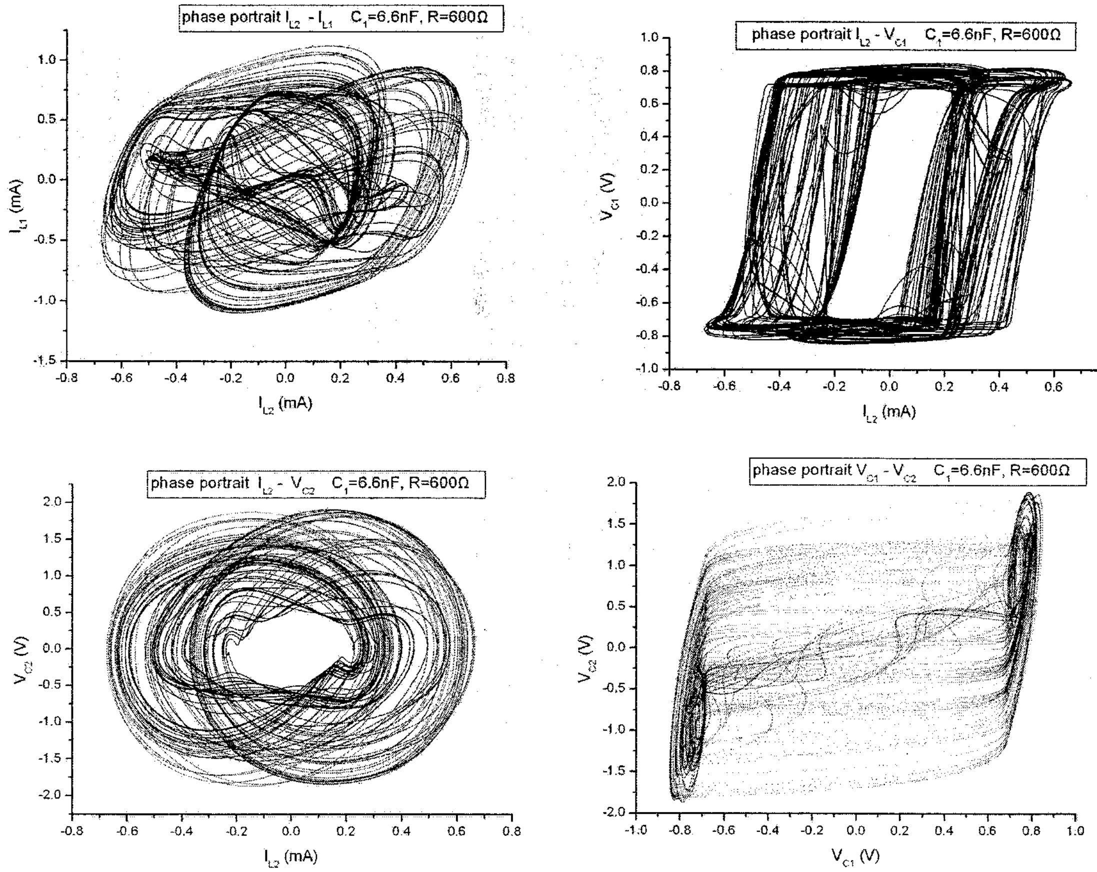

Figure 15. a,b,c,d. Phase Portraits for R=600 Ω, C1=6.6nF, LE1=0.034>0, LE2=0,LE3=-0.10(chaos),

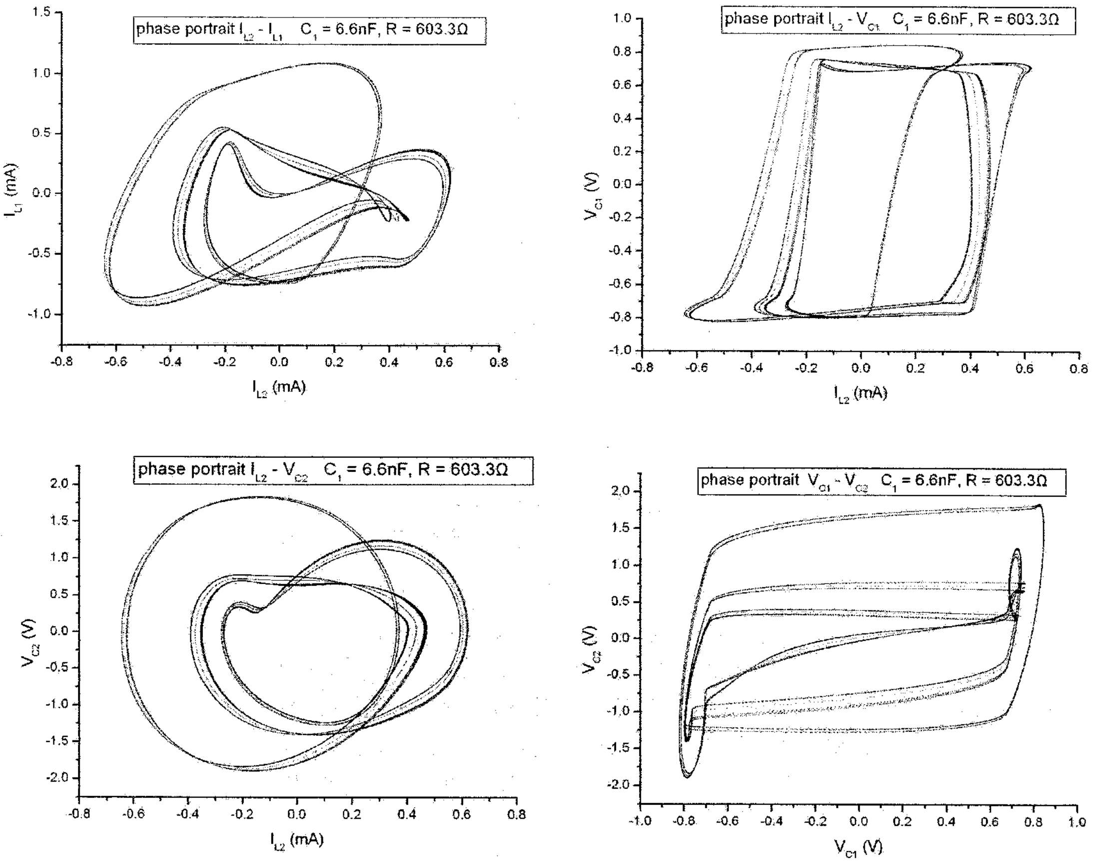

Figure 16. a,b,c,d. Phase Portraits for R=603.3 Ω, C_1=6.6nF, LE1=0, LE2=-0.031<0,LE3=-0.121 (periodic), p-12,

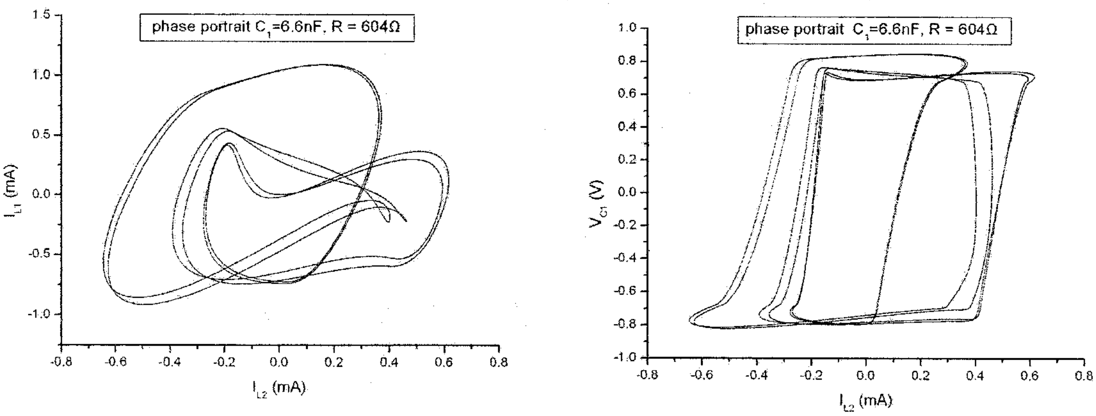

Figure 17. a,b. Phase Portraits for R=604 Ω, C_1=6.6nF,

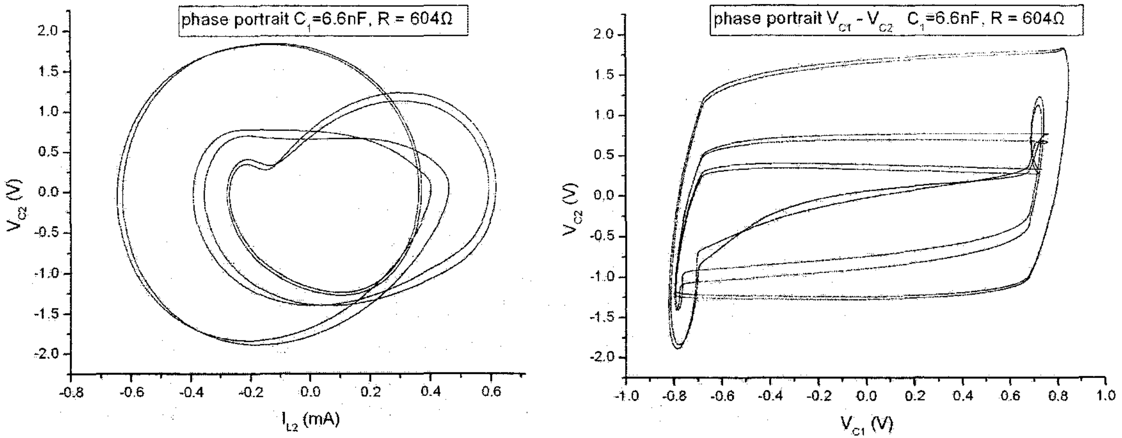

Figure 17. c,d. Phase Portraits for R=604 Ω, C_1=6.6nF,
LE1=0, LE2=-0.0306<0,LE3=-0.116 (periodic), p-6.

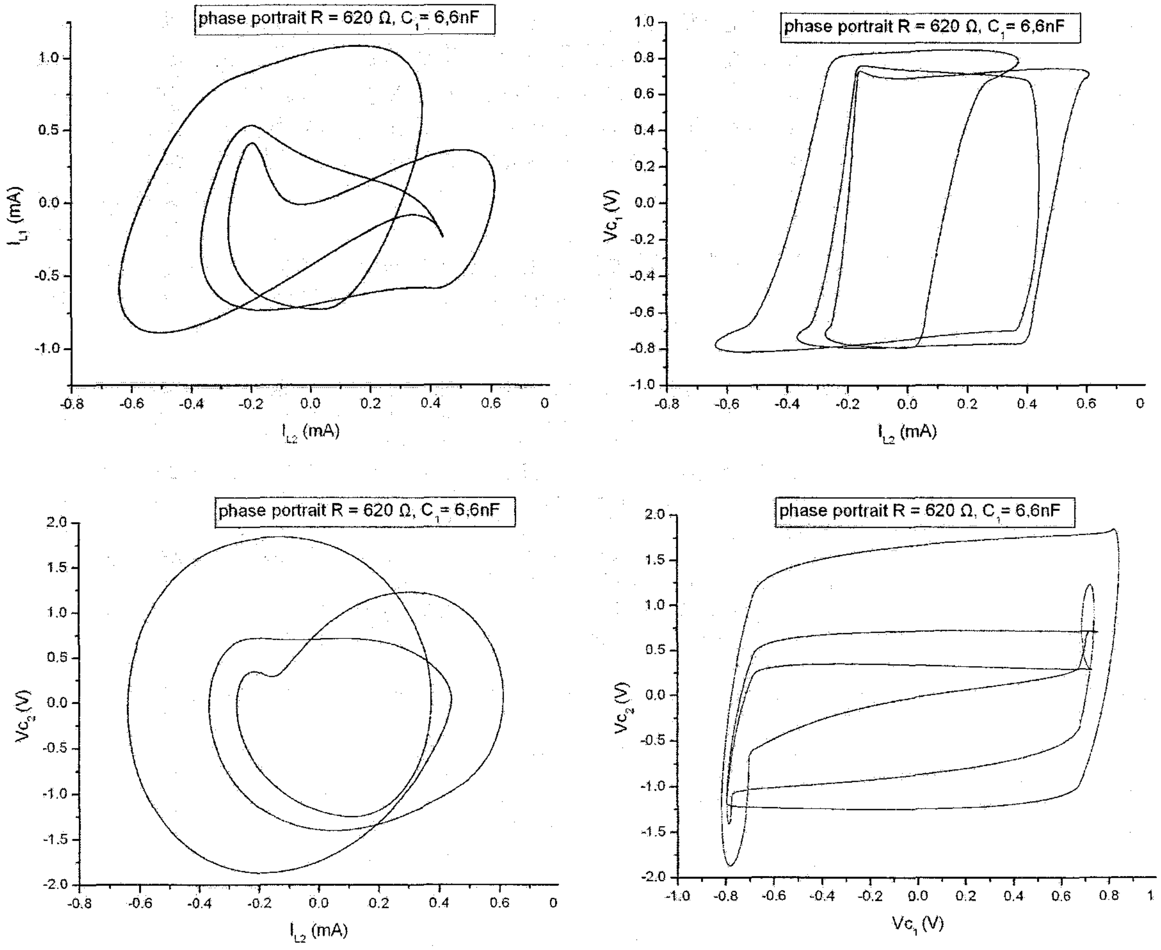

Figure 18. a,b,c,d. Phase Portraits for R=620 Ω, C_1=6.6nF,
LE1=0, LE2=-0.069<0,LE3=-0.079 (periodic), p-3.

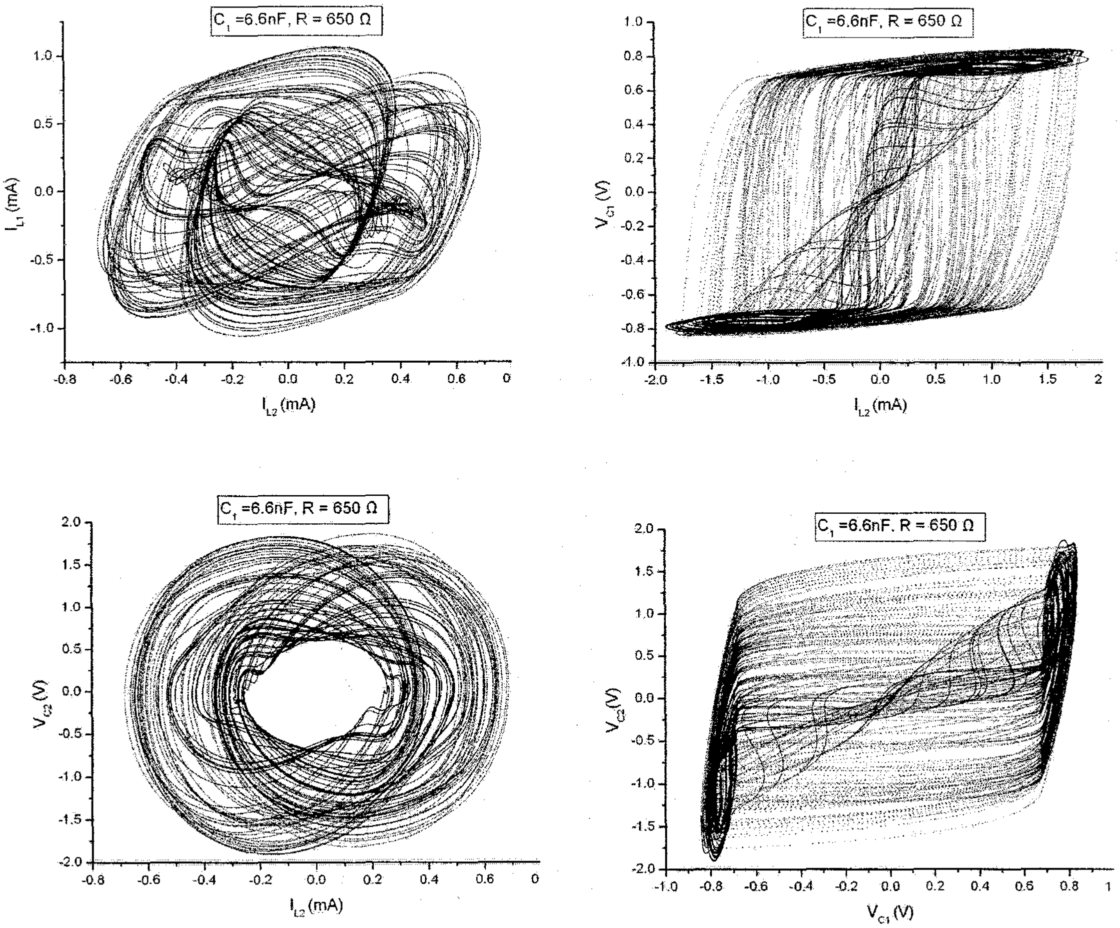

Figure 19. a,b,c,d. Phase Portraits for R=650 Ω, C_1=6.6nF, LE1=0.0466>0, LE2=0,LE3=-0.112(chaos).

⇨R_1 = 600Ω (chaos),	Figure 15a,b,c,d.	
⇨R_1 = 603.3Ω / p – 12,	Figure 16a,b,c,d.	
⇨R_1 = 604Ω / p – 6,	Figure 17a,b,c,d.	
⇨R_1 = 620Ω / p - 3 a,	Figure 18a,b,c,d.	and
⇨R_1= 650Ω (chaos),	Figure 19a,b,c,d.	

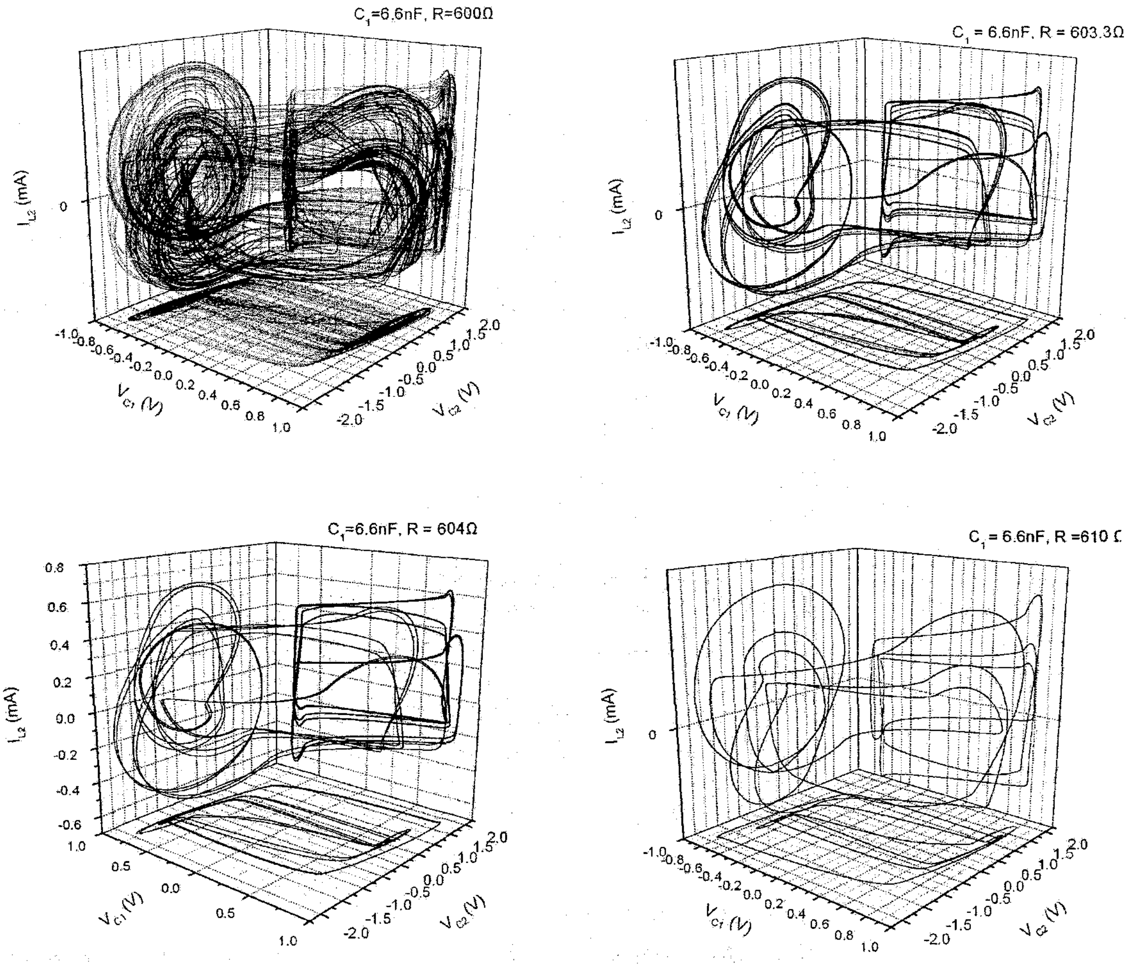

Figure 20. a,b,c,d. 3D $V_{C1} - V_{C2} - I_{L2}$, C_1=6.6nF for different R_1.

In Figures (15a,b,c,d – 19a,b,c,d} we have shown the phase portrait diagrams for discrete values of the resistor R_1. In Figures (20a,b,c,d) we presence the phase portraits diagrams for the same values of the resistor R_1 (0.600, 0.6033, 0.604, 0.610kΩ). These diagrams show the development of the system into the phase space for a route to chaos via period doubling. In Figure 20d we have for R_1 = 610Ω / period – 3. In Figure 20c we have for R_1 = 604Ω / period – 6, for R_1 = 603.3Ω / period – 12 (Figure 20b) and finally in Figure 20a chaos.

In 3-D diagrams we also observe the projection of the trajectory to different planes. The 3-D diagrams give us more exactly information for the dynamical behavior of the system. Although in simple phase portrait diagrams, some trajectories seem to be intercepted, the description in 3-D diagrams makes clear that they are just in different levels.

3. Conclusions

❖In this paper we have studied the dynamic behavior of an autonomous electric circuit of 4^{th} order with two active elements, one linear negative conductance (G = -0.45mS) and one non linear resistor of N-type with a symmetrical piecewise linear v-i characteristic (Ga=-0.105mS, Gb= 7mS). Using the resistor R_1 as the control parameter we have observed antimonotonicity in a narrow region of R_1 values for C_1=6.6nF.

❖In the bifurcation diagram (Figure 5), formation of ''bubbles'' has been observed for some initial conditions (i.c.1). This scheme called "period –3– bubble". As the value of R_1 is increased the system remains in periodic state and no chaotic state appear. For other initial conditions (i.c.2) the behavior of the system is completely different. In the same region of R_1 values we observe (Figure 6) alternation between period and chaotic states. The creation of the "bubbles" is very sensitive to initial conditions.

❖We have also observed routes to chaos such us from periodic to quasi-periodic and finally to chaos. A route to chaos via a reverse period doubling in another range changing of R_1 (0.600 – 0.650kΩ) appear (Figure 15a,b,c,d. - Figure 19a,b,c,d.).

❖ Using the value of the resistor R_1 as the control parameter we have varied the coupling between the resonance circuit and the branch of the non linear resistor.

❖Finally we have studied the dependence of circuit behavior on the initial conditions. We have periodic state for values of R_1 equal to 0.737kΩ and 0.742kΩ (Figure 6 - Figure 8) with initial conditions 1, but the state of the system varies to quasiperiodic (Figure 7 - Figure 9), while the initial conditions are changing (i.c.2.). The coexistence at least two different attractors, drive the circuit in two different ways to chaos, but for smaller values of R_1 we have convocation in the same way.

Acknowledgements

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