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An Investigation on the Janowskı Convex Function of Complex Order

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Abstract
In this paper We shall give some results for the Janowski convex function of complex order.

Özet
Bu makalede kompleks mertebeden Janowski konveks fonksiyonları için bazı neticeler veririz.

Key words and pharses: *Convex function of complex order, Starlike function of complex order. A.M.S Subject classification (2002) primary 30C45*

I. Introduction

Let Ω be the family of functions $\Psi(z)$ regular in the unit disc D= {z...| ...| z| <1 }, and satifying the conditions $\Psi(0) = 0, |\Psi(z)| <1$ for $z \varepsilon$ D.

Next, fòr arbitrary fixed numbers A, B, $-1 \leq B < A \leq 1$ denote by P(A,B) the family of functions $p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots$ regular in D such that is in P(A,B) if and only if

$$p(z) = \frac{1 + A\Psi(z)}{1 + B\Psi(z)}$$

for some function $\Psi(z)\ \varepsilon\ \Omega$ and every $z \in$ D.

$$f(z) = z + a_2 z^2 + a_3 z^3 + \ldots,\ f'(z) \neq 0$$

regular in D and such that $f(z)$ is in C(A,B,b) if and only if

$$1 + \frac{1}{b} z.\frac{f''(z)}{f'(z)} = p(z)$$

for some $p(z) \in$ P(A,B) and all $z \in$ D

Finally,we consider the following class of functions which are regular in D and

$$f(z) = z + a_2 z^2 + a_3 z^3 + \ldots\ ,\frac{f(z)}{z} \neq 0, 1 + \frac{1}{b}(z.\frac{f'(z)}{f(z)} - 1) = p(z) \text{ for some}$$

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$p(z) \in$ P(A,B) and for all z in D.This class is denoted by $S^*(A,B,b)$.

We note that P(1,-1) is the class of Caratheodory functions.Therefore C(A,B,b) contains the following classes.

1) C(1,-1,1) is the well known class of convex functions [1].
2) C(1,-1,b) is the class of convex functions of complex order.This class was introduced by P.Wiatrowski [8],and M.K.Nasr and M.K.Aouf [3] , [4]
3) C(1,-1,1-β), $0 \leq \beta < 1$ is the class of convex funcyions of order β.This class was introduced by M.S.Robertson [5]
4) C(1,-1,$e^{-i\lambda}.Cos\lambda$), $|\lambda| < \frac{\pi}{2}$ is the class of functions for which $z.f'(z)$ is $\lambda-$Spirallike functions.This class was introduced by M.S.Robertson [6].
5) C(1,-1,$(1-\beta).e^{-i\lambda}.Cos\lambda$), $0 \leq \beta < 1, |\lambda| < \frac{\pi}{2}$ is the class of functions for which $z.f'(z)$ is $\lambda-$Spirallike of order β.This class was introduced by P.I.Sizuk [7]

İf we write $CT(b) \quad = 1 + \frac{1}{b}.z.\frac{f''(z)}{f'(z)}$

6) C(1,0,b) is the set defined by $|CT(b) - 1| < 1$
7) C(β,0, b), $0 \leq \beta < 1$ is the set defined by $|CT(b) - 1| < \beta$
8) C($\beta,-\beta$, b), $0 \leq \beta < 1$ is the set defined by $\left|\frac{CT(b)-1}{CT(b)+1}\right| < \beta$
9) C(1, $(-1 + \frac{1}{M})$, b), $M \geq 1$ is the set defined by $|CT(b) - M| < M$
10) C($1 - 2\beta, -1$, b), $0 \leq \beta < 1$ is the set defined by $\operatorname{Re} CT(b) \geq \beta$

II. Auxiliary Lemmas.

In this section of this paper we shall give the following lemmas for the purpose of this paper.

Lemma 2.1 Let $f(z) \in$ C(A,B,b), then the function $g(z) = z.f'(z)$ belongs to $S^*(A,B,b)$

Proof. If we take the logarithmic derivative from the equality $g(z) = z.f'(z)$ we get

$$\textbf{(2.1)} \qquad z.\frac{g'(z)}{g(z)} = 1 + z.\frac{f''(z)}{f'(z)} \,.$$

After the simple calculations from the equality (2.1) we obtain

$$\textbf{(2.2)} \qquad 1 + \frac{1}{b}(z.\frac{g'(z)}{g(z)} - 1) = 1 + \frac{1}{b}.z\frac{f''(z)}{f'(z)} \,.$$

The equality (2.2) shows that this lemma is true.

Lemma 2.2. Let $f(z) \in C(A, B, b)$ then the derivative of $f(z)$ is given by the relation

$$f'(z) = \begin{cases} (1 + B\Psi(z))^{\frac{b(A-B)}{B}} & B \neq 0 \\ e^{bA\Psi(z)} & B = 0 \end{cases}$$

Where $\Psi(z) \in \Omega$

Proof:

Step one:Let $B \neq 0$ and $f'(z) = (1 + B\Psi(z))^{\frac{b(A-B)}{B}}$.

İf we take the logarithmic derivative from this equality and a simple calculations we obtain

$$\textbf{(2.3)} \qquad z\frac{f''(z)}{f(z)} = b(A-B).\frac{z.\Psi'(z)}{\Psi(z)} .$$

By using I.S.Jack's Lemma [2] in the equality (2.3) and a simple calculations shows that

$$\textbf{(2.4)} \qquad 1+\frac{1}{b}.z\frac{f''(z)}{f'(z)}=\frac{1+A\Psi(z)}{1+B\Psi(z)}.$$

The equality (2.4) shows that $f(z)\in C(A,B,b)$.

Step two: Let $B=0$,and $f'(z)=e^{b(A-B)}$.Similarly we obtain

$$\textbf{(2.5)} \qquad 1+\frac{1}{b}z.\frac{f''(z)}{f'(z)}=\frac{1+A\Psi(z)}{1+B\Psi(z)}$$

the equality (2.5) shows that $f(z)\in C(A,B,b)$

Corollary.2.1. From the Lemma 2.1 and Lemma2.2 we obtain that the function

$$f(z)=\begin{cases}\int_0^z (1+B\zeta)^{\frac{b(A-B)}{B}}\,d\zeta & B\neq 0\\ \int_0^z e^{bA\zeta}\,d\zeta & B=0\end{cases}$$

belongs to $C(A,B,b)$

Theorem.2.1 The set C(A,B,b) is invariant under the rotation,so that $e^{-i\alpha}f(e^{i\alpha}z)$ is in C(A,B,b) whenever $f(z)$ is in C(A,B,b)

Proof: Let $f(z)$ is in C(A,B,b) then the equality

$$\textbf{(2.6)} \qquad 1+\frac{1}{b}.z.\frac{f''(z)}{f'(z)}=\frac{1+A\Psi(\zeta)}{1+B\Psi(\zeta)}$$

is satisfied. Where $\Psi(\zeta)\in\Omega$.On the other hand if we write

$$(*) \qquad g(z)=e^{-i\alpha}f(e^{-i\alpha}z)\ ,$$

and we take the logarithmic derivative from the relation (*) and a simple calculations shows that

$$(2.7) \qquad 1+\frac{1}{b}.z.\frac{g''(z)}{g'(z)}=1+\frac{1}{b}.e^{i\alpha}.z.\frac{f''(e^{i\alpha}z)}{f'(e^{i\alpha}.z)}.$$

Now if we take $\zeta = e^{i\alpha}.z$ then we have

$$(2.8) \qquad |\zeta| = |e^{i\alpha}.z| = |e^{i\alpha}|.|z| \le 1.|z| < 1$$

Consider the relations (2.6),(2.7) and (2.8) together we obtain that

$$(2.9) \qquad 1+\frac{1}{b}.z.\frac{g''(z)}{g'(z)}=\frac{1+A\Psi(z)}{1+B\Psi(z)}.$$

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