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DYNAMICS OF TWO COUPLED, NONLINEAR, AUTONOMOUS 4th ORDER CIRCUITS

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Abstract

In this paper we have studied the case of chaotic synchronization of two identical, mutually coupled, non linear, 4th order autonomous electric circuits. Each circuit contains one linear negative conductance G and one non linear resistor R_N with a symmetrical piecewise-linear υ–i characteristic. Using the capacitances C_1 and C_2 as the control parameters, we have observed the transition from periodic states to chaotic ones and vise versa, as well as crisis phenomena. We have established the mutual coupling via a variable resistor R_x. We have observed that synchronization is possible as the resistance of the coupling element is varied.

Keywords: *Chaotic synchronization, Autonomous circuit, Control parameter, Crisis, Mutual coupling.*

1. Introduction

Since the discovery by Pecora and Carroll [1], that chaotic systems can be synchronized, the topic of synchronization of coupled chaotic circuits and systems has been studied intensely [2] and some interesting applications, as in broadband communications systems, have come out of this research [3-5]. Generally, there are two methods of chaos synchronization available in the literature. In the first method, due to Pecora and Carroll [1], a stable subsystem of a chaotic system could be synchronized with a separate chaotic system, under certain suitable conditions. The second method to achieve chaos synchronization between two identical nonlinear systems is due to the effect of resistive coupling without requiring to construct any stable subsystem [6-8]. According to Carroll and Pecora [9], periodically forced synchronized chaotic circuits are much more noise-resistant than autonomous synchronized chaotic circuits.

In this paper we have studied the case of two identical fourth-order autonomous nonlinear electric circuits (Figure 1) with two active elements, one linear negative conductance and one nonlinear resistor with a symmetrical piecewise linear v - i characteristic (Figure 2). Using the capacitances C_1 and C_2 as the control parameters, we have observed a reverse period-doubling sequence, as well as a crisis phenomenon, when the spiral attractor suddenly widens to a double-scroll attractor [10].

2. The Non Linear 4th Order Circuit

The circuit, we have studied, is shown in Figure 1 and has two active elements, a nonlinear resistor R_N and a linear negative conductance G, which were implemented using the circuits shown in Figures 3 and 4 respectively. The v-i characteristic of the negative conductance G is shown in Figure 5.

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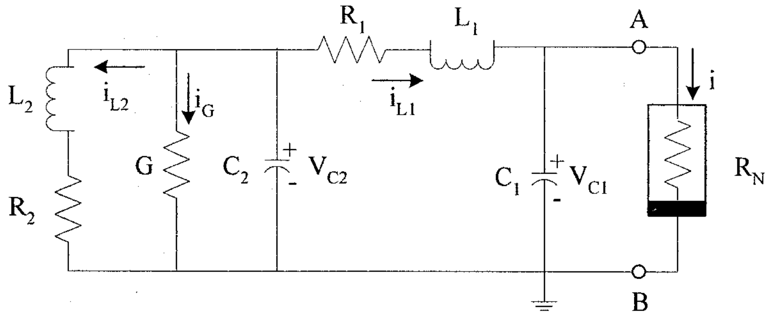


Figure 1. The 4th order autonomous circuit

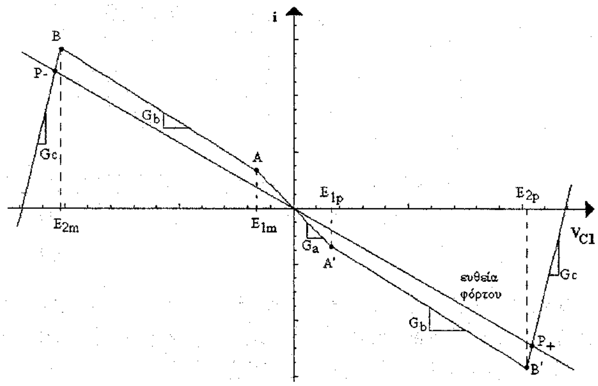


Figure 2. The piece-wise linear and symmetrical v-i characteristic of nonlinear resistor R_N

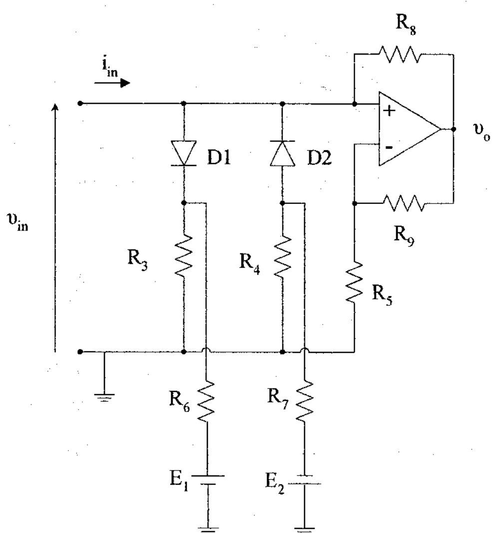


Figure 3. The nonlinear resistor R_N implementation

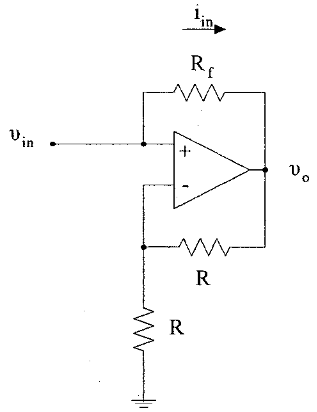


Figure 4. The negative conductance implementation

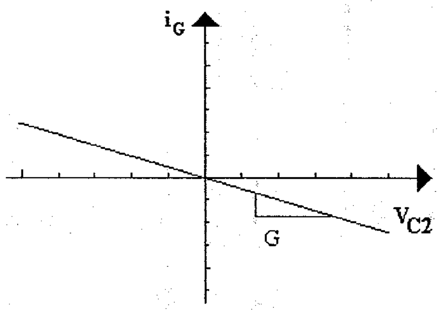


Figure 5. The v-i characteristic of negative conductance

The state equations of the circuit are:

$$\frac{d\upsilon_{C1}}{dt} = \frac{1}{C_1} \cdot \{ i_{L1} - f(\upsilon_{C1}) \} \tag{2.1}$$

$$\frac{d\upsilon_{C2}}{dt} = -\frac{1}{C_2} \cdot \{ G \cdot \upsilon_{C2} + i_{L1} + i_{L2} \} \tag{2.2}$$

$$\frac{di_{L1}}{dt} = \frac{1}{L_1} \cdot \{ \upsilon_{C2} - i_{L1} R_1 - \upsilon_{C1} \} \tag{2.3}$$

$$\frac{di_{L2}}{dt} = \frac{1}{L_2} \cdot \{ \upsilon_{C2} - i_{L2} R_2 \} \tag{2.4}$$

where:

$$i = G_c \upsilon_{C1} + 0.5(G_a - G_b)\cdot\left(\left|\upsilon_{C1} + E_{1p}\right| - \left|\upsilon_{C1} - E_{1p}\right|\right) + 0.5(G_b - G_c)\cdot\left(\left|\upsilon_{C1} + E_{2p}\right| - \left|\upsilon_{C1} - E_{2p}\right|\right) \quad (2.5)$$

The values of circuit's parameters are: L_1=10.2mH, L_2=21.5mH, R_1=2.02KΩ, R_2=108Ω, G=-0.5mS, G_a=-0.833mS, G_b=-0.514mS, G_c=2.9mS, E_{1p}=1.46V and E_{2p}=10.1V.

3. Mutual Coupling of Two Identical Nonlinear 4th Order Circuits

If two identical circuits are resistively coupled, complex dynamics can be observed, as well as chaotic synchronization, as the coupling parameter R_X is varied. The two circuits can be either υ_{C1}-coupled or υ_{C2}-coupled. The system of the two identical 4th order nonlinear circuits υ_{C1}-coupled via the linear resistor R_X is shown in Figure 6. The state equations of the coupled system are:

$$\frac{d\upsilon_{C11}}{dt} = -\frac{1}{C_1}\cdot\left\{\frac{\upsilon_{C11} - \upsilon_{C12}}{R_x} - i_{L11} + f(\upsilon_{C11})\right\} \quad (3.1)$$

$$\frac{d\upsilon_{C21}}{dt} = -\frac{1}{C_2}\cdot\left\{G\cdot\upsilon_{C21} + i_{L11} + i_{L21}\right\} \quad (3.2)$$

$$\frac{di_{L11}}{dt} = \frac{1}{L_1}\cdot\left\{\upsilon_{C21} - i_{L11}R_1 - \upsilon_{C11}\right\} \quad (3.3)$$

$$\frac{di_{L21}}{dt} = \frac{1}{L_2}\cdot\left\{\upsilon_{C21} - i_{L21}R_2\right\} \quad (3.4)$$

$$\frac{d\upsilon_{C12}}{dt} = -\frac{1}{C_1}\cdot\left\{-\frac{\upsilon_{C11} - \upsilon_{C12}}{R_x} - i_{L12} + f(\upsilon_{C11})\right\} \quad (3.5)$$

$$\frac{d\upsilon_{C22}}{dt} = -\frac{1}{C_2}\cdot\left\{G\cdot\upsilon_{C22} + i_{L12} + i_{L22}\right\} \quad (3.6)$$

$$\frac{di_{L12}}{dt} = \frac{1}{L_1}\cdot\left\{\upsilon_{C22} - i_{L12}R_1 - \upsilon_{C12}\right\} \quad (3.7)$$

$$\frac{di_{L22}}{dt} = \frac{1}{L_2}\cdot\left\{\upsilon_{C22} - i_{L22}R_2\right\} \quad (3.8)$$

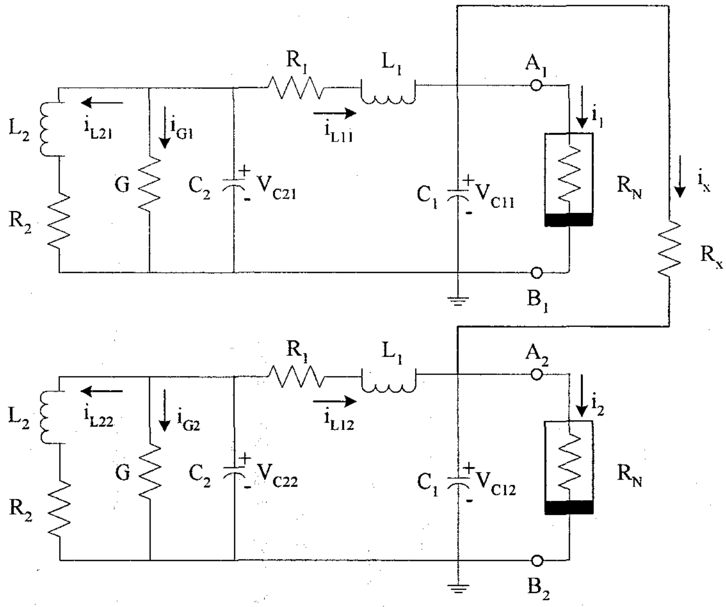


Figure 6. The υ_{C1}-coupled system of the two identical 4th order nonlinear circuits

4. Experimental Results

4.1Dynamic behavior of the 4th order nonlinear circuit

The chaotic attractors of the two circuits, when they are uncoupled, are shown in the following figures. In Figures 7-14 we can see the portraits x-y → i_{L2}-υ_{C2} for C_2=4.2nF. In Figure 7, we see i_{L2} vs. υ_{C2} for C_1=5.8nF and we observe that the system is in periodic state. In Figure 8, the i_{L2} vs. υ_{C2} diagram is presented for C_1=6.7nF and we observe the double-scroll attractor. In Figures 9-11, the phase portraits are shown for C_1=7.3nF, C_1=7.4nF and C_1=7.5nF respectively, and we observe the chaotic attractors. For C_1=7.6nF, C_1=9.2nF and C_1=9.6nF (Figures 12-14 respectively) the system is in periodic states with periods 4-2-1, respectively.

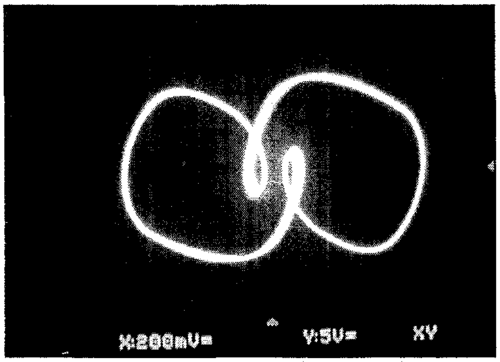


Figure 7. i_{L2} vs. v_{C2} for C_2=4.2nF and C_1=5.8nF

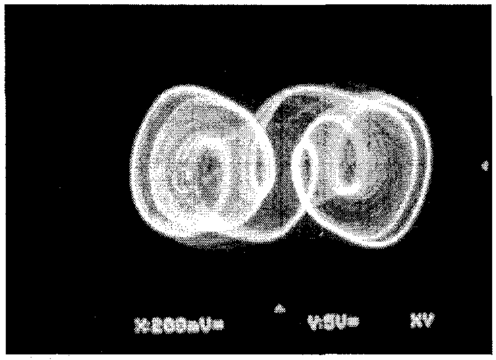


Figure 8. i_{L2} vs. v_{C2} for C_2=4.2nF and C_1=6.7nF

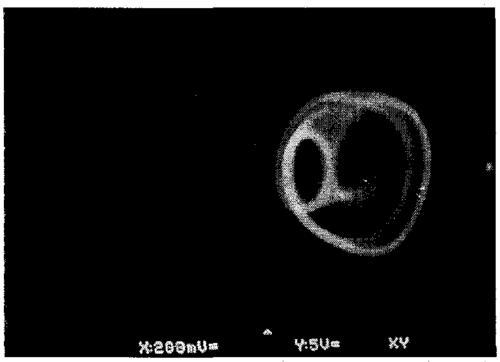


Figure 9. i_{L2} vs. v_{C2} for C_2=4.2nF and C_1=7.3nF

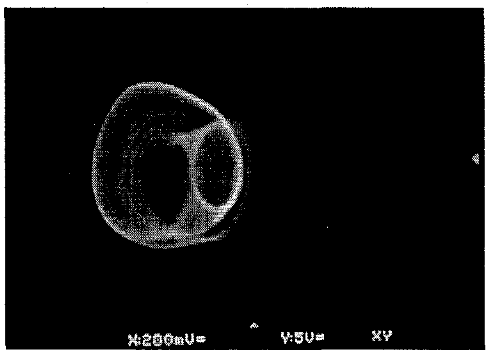


Figure 10.i_{L2} vs. v_{C2} for C_2=4.2nF and C_1=7.4nF

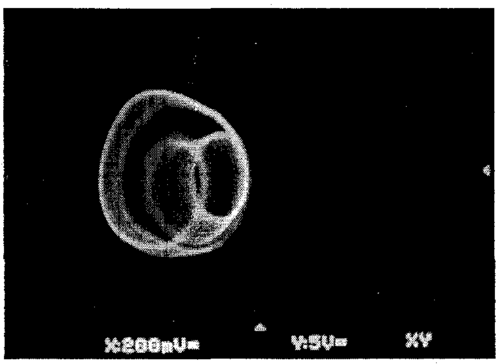


Figure 11.i_{L2} vs v_{C2} for C_2=4.2nF and C_1=7.5nF

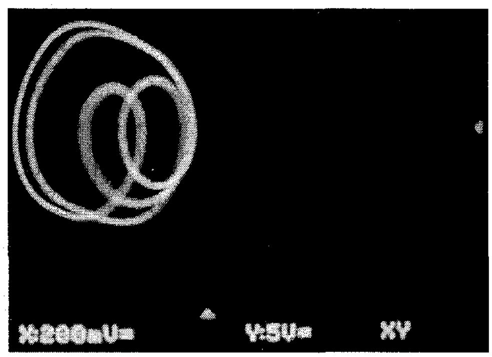


Figure 12.i_{L2} vs v_{C2} for C_2=4.2nF and C_1=7.6nF

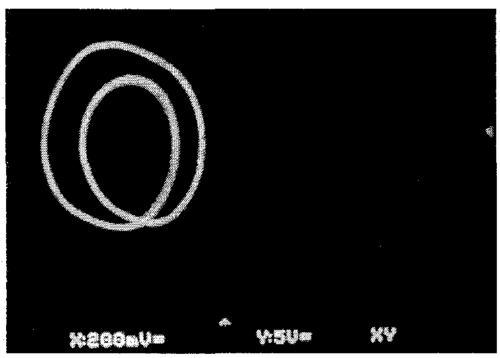


Figure 13.i_{L2} vs v_{C2} for C_2=4.2nF and C_1=9.2nF

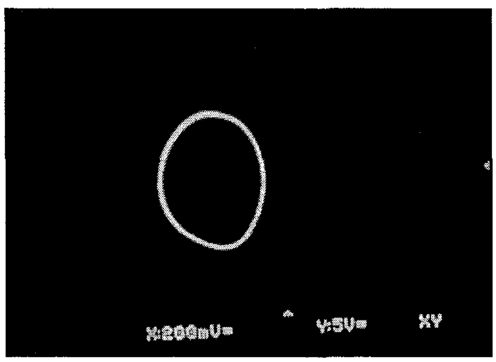
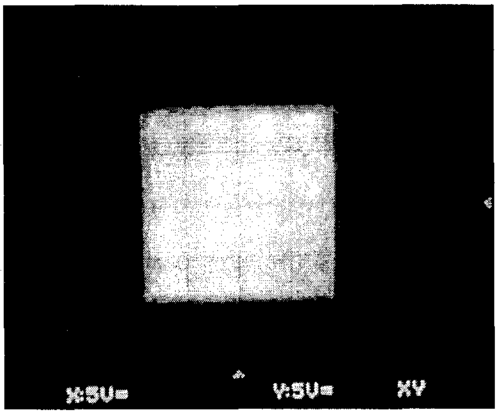


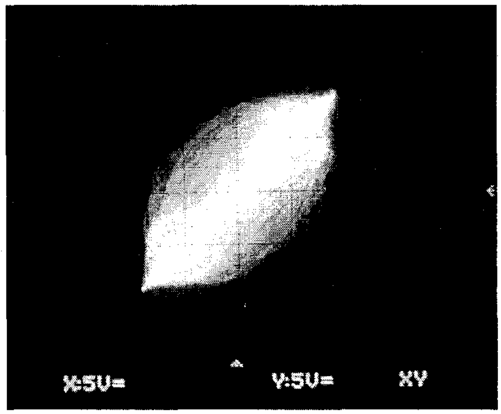
Figure 14. i_{L2} vs. v_{C2} for C_2=4.2nF and C_1=9.6nF

Coupling two identical 4th order nonlinear circuits

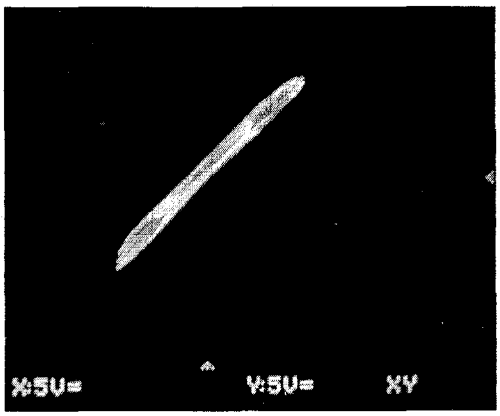
We have studied the possibility of chaotic synchronization when the initial conditions of the coupled circuits belong to the same basin of attraction. In Figures 15(a)-(d) we can see the experimental results from the coupled system for C_2=4.20nF, C_1=6.7nF and various values of the coupling resistor R_x. The diagrams are x-y → υ_{C21}-υ_{C22}. We observe how the system passes from nonsynchronized states to synchronized ones, depending on the coupling resistance R_x.



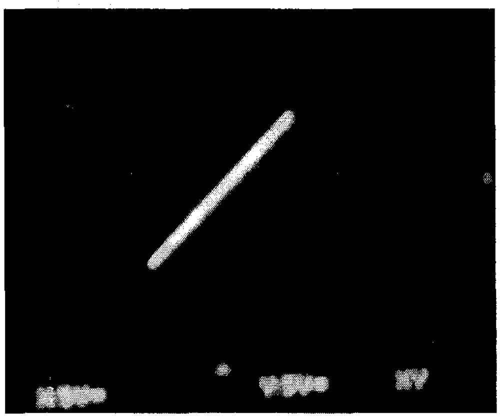
(a)



(b)



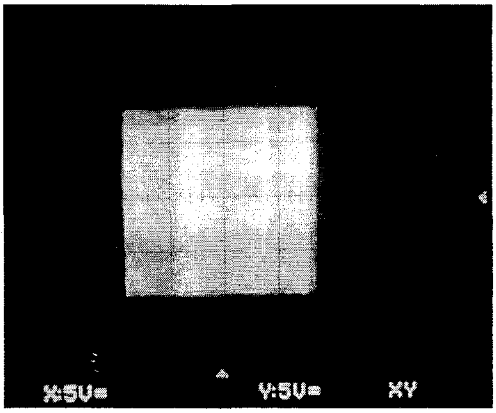
(c)



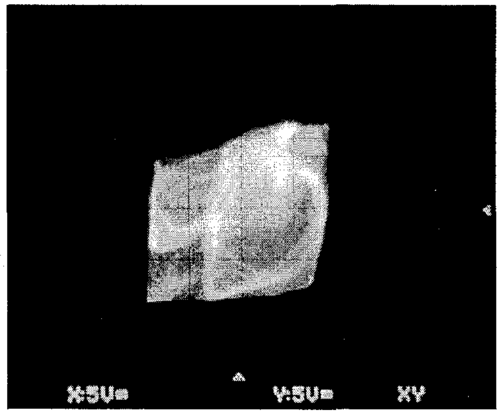
(d)

Figure 15. v_{C21} vs. v_{C22} for C_1=6.7nF, C_2=4.2nF and (a) R_x= ∞(b) R_x=11KΩ (c) R_x=1.1KΩ (d) R_x= 99Ω

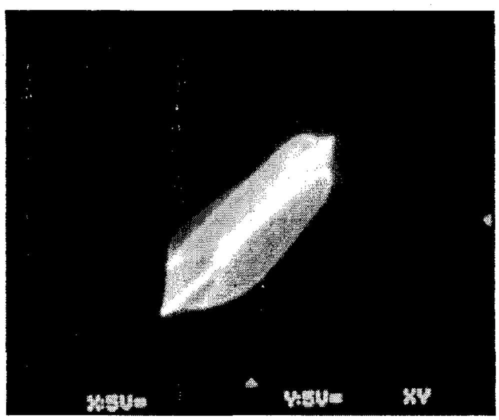
In Figures 16(a)-(d) we can see the experimental results from the coupled system for C_2=4.20nF, C_1=7.3nF and various values of the coupling resistor R_x. The diagrams are x-y → v_{C21}-v_{C22}. We also observe how the system passes from nonsynchronized states to synchronized ones, as the coupling resistance R_x varies.



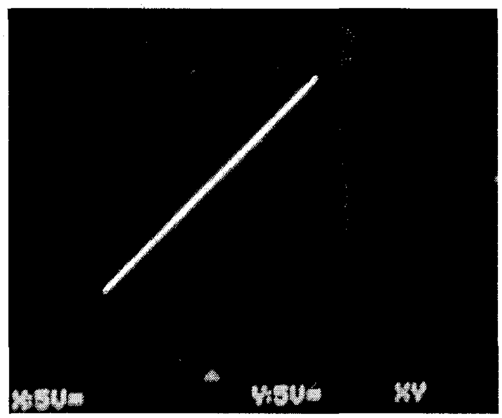
(a)



(b)



(c)



(d)

Figure 16. v_{C21} vs. v_{C22} for C_1=7.3nF, C_2=4.2nF and (a) $R_x = \infty$ (b) R_x= 30KΩ (c) R_x= 8KΩ (d) R_x=1KΩ

5. Conclusion

In this paper we have implemented two identical 4th order circuits and we have shown that chaotic synchronization is possible experimentally.

We have studied the dynamics of the individual nonlinear autonomous electric circuit.

Using the capacitances C_1 and C_2 as the control parameters, we have observed the transition from periodic states to chaotic ones and vise versa, as well as crisis phenomena, when the spiral attractor suddenly widens to a double-scroll attractor (Figures 9 and 10).

Furthermore, we have seen the chaotic synchronization of two identical circuits, which are mutually coupled via a linear resistor R_x, when the initials conditions of the coupled circuits belong to the same basin of attraction. Experimental results have shown that chaotic synchronization is possible in the case of υ_{C2}-coupling. We have observed that for C_2=4.2nF and C_1=6.7nF chaotic synchronization is possible for coupling parameter $\epsilon=R_1/R_x>20.4$, while for C_2=4.2nF and C_1=7.3nF is possible for $\epsilon>2.02$. So we conclude that the capacitance C_1 and the coupling parameter ϵ are conversely proportional quantities.

Acknowledgments

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List of Figures: