Journal of İstanbul Kültür University
2002/1. pp. 1-4.

A Coefficient Inequality for Convex Functions

Yaşar Polatoğlu *

Abstract
In this study an important result of the paper called' A characterization for convex functions of complex order'(Ist. Üniv. Fen Fak. Matematik Dergisi cilt 54 sayfa 175- 179, 1997) is given and we present a coefficient inequality for convex functions under the regularly univalent conditions.

Özet
Biz bu makalede'A characterization for convex functions of complex order (Ist. Üniv. Fen Fak. Matematik Dergisi cilt 54 sayfa 175-170, 1997) adlı makalenin çok önemli bir neticesi olan katsayı eşitsizliğini veririz.

Keywords : Coefficient inequality, λ -Spirallike functions,Convex function of complex order.

Introduction:

Let R denote the class of functions

$$f(z) = z + a_2 z^2 + a_3 z^3 + \ldots.$$

which are analytic in the unit disc $D = \{z \ / \ |z| \le 1 \}$

A function $f(z)$ in R, is said to be a convex function of complex order b ($b \neq 0$,complex)that is $f(z) \in C(b)$ if and only if $f'(z) \neq 0$, and

$$\text{Re } (1 + \frac{1}{b} z.\frac{f''(z)}{f'(z)}) \ge 0, z \in D$$

The class C(b) was introduced by P.Wiatrowski [3]. By giving specific values to b , we obtain the following important subclasses:

(i) C(1) is a well known class of convex functions,

(ii) $C(1-\beta)$, $0 \le \beta \le 1$ is the class of convex functions of order β,

(iii) $C(e^{-i.\lambda}.Coc\lambda).|\lambda| \le \frac{\pi}{2}$ is the class of functions for whicw $z.f'(z)$ i λ Spirallike,

(iv) $C((1-\beta).e^{-i.\lambda}.Cos\lambda), 0 \le \beta < 1, |\lambda| < \frac{\pi}{2}$ is the class of functions for which $z.f'(z)$ **is** λ – Spirallike of order β $[See. 1,3,4,5]$.

*Department of Mathematics İstanbul Kültür Üniversity 34510 Şirinevler İstanbul

Theorem 1.1.
Let

$$f(z)=z+a_2z^2+a_3z^3+\dots$$

be analytic in D. A necessary and sufficient condition that

$$f(z)\in C(b)$$

is for each real number $k,..-1<k<1$,the functions $F(k,b,z,\eta)$ defined by the equations, is

$$\textbf{(1.2)}\qquad F(k,b,z,\eta)=\left[\frac{k(f(z)-f(\eta))}{f(kz)-f(k\eta)}\right]^{\frac{1}{b}}$$

$$\textbf{(1.3)}\qquad F(k,b,0,0)=1$$

$$\textbf{(1.4)}\qquad F(1,b,z,\eta)=\left[\frac{f(z)-f(\eta)}{z-\eta}\right]^{\frac{1}{b}}$$

analytic and subordinate to

$$P(z)=\frac{1+kz}{1+z},..z\in D$$

or equivalently that

$$\textbf{(1.5)}\qquad \operatorname{Re}F(k,b,z,\eta)\geq\frac{1+k}{2},\left|\frac{1+k}{F(k,b,z,\eta)}-1\right|<1$$

Definition:
Let $f(z)$ satisfies the inequality

$$\left|\frac{f(z)-f(\eta)}{z-\eta}\right|>m,m>0,z\in D,\eta\in D$$

then $f(z)$ is called regularly in D [2].

Coefficient Inequality For Convex Function

In this section we shall give a coefficient inequality for convex function under the regularly univalent condition.

Now we consider the inequality (This inequality is dotained from the (1.5) for k=0,b=1)

$$\textbf{(2.1)}\qquad \operatorname{Re}F(0,1,z,\eta)=\operatorname{Re}\left[\frac{f(z)-f(\eta)}{z-\eta}\right]>\frac{1}{2}$$

on the other hand, the function

$$F(0,1,z,\eta)$$

is analytic and continous in D; therefore, we have

(2.2)
$$\lim_{z\leftarrow\eta} \operatorname{Re}(F(0,1,z,\eta)) = \lim_{z\to\eta}\left[\operatorname{Re}\frac{f(z)-f(\eta)}{z-\eta}\right]$$
$$= \operatorname{Re}(\lim_{z\leftarrow\eta}\frac{f(z)-f(\eta)}{z-\eta}) = \operatorname{Re}(f(z) > \frac{1}{2}$$

(2.3) $P(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots$

is analytic in D and satisfies $P(0)=1, \operatorname{Re} P(z) > 0$ then $|p_n| \leq 2$. These functions are called Caratheodory functions. Considering the relations (2.2) and (2.3) together, we get

(2.4) $P(z) = 2.f(z) - 1$

from the relation (2.4) we have

(2.5) $2.n.a_n = p_n$

if we use Caratheodory inequality $|p_n| \leq 2$ in the equality (2.5), we obtain

(2.6) $|a_n| \leq \dfrac{1}{n}$

The inequality (2.6) is a new inequality for convex functions under the regularly univalent condition. This inequality is sharp because the function

$$f_*(z) = Log\frac{1}{z-1} = z + \frac{1}{2}z^2 + \frac{1}{3}z^3 + \ldots + \frac{1}{n}z^n + \ldots$$

is an extremal function and this function satisfies

$$\left|\frac{f_*(z)-f_*(\xi)}{z-z.\xi}\right| = \left|\frac{Log\dfrac{1-z.\xi}{1-z}}{z-z.\xi}\right| \neq 0 \quad ,|z|<1, |\xi|<1, z\neq\xi$$

Therefore, the condition of regularly univalent is satisfied by this function.

References

[1] Goodman, A.W., (1983), "*Univalent Functions*",*Volume.I and VolumeII*, Tampa Florida, II Mariner Comp.

[2] Alisbah,O.H., (1948), "Uber starksclichte Abdilung des Einheitkrises", Universite d'İstanbul Faculte des Sciences.Recueil de Memories Commenorant la pose de la premiere des Nouveaux Instituts des Sciences, İstanbul University, 39-44.

[3] Wiatrowski, P., (1971) , "The coefficiet of certain family of holomorphic functions",Nauk.Univ.Todzk.Nauki.Math Przyord ser II.Zesty (39) Math. 57-85

[4] Polatoğlu, Y., (1997),"A characterization for convex function of complex order b.", İst.Üniv.Fen-Fak.Matematik Dergisi Cilt 54 , 175-197.

[5] Polatoğlu, Y., (1995), "Radius problem for convex functions of complex order", Tr.J.of Mathematics ,19, 1-7.

Journal of İstanbul Kültür University
2002/1, pp. 75-80.

Optimization Scheme of Offshore Steel Structures

Nıjat MASTANZADE*

Abstract:
The finite elements method, Reyleigh correlation and Lagrange multipliers method for the problem of optimization of offshore platform. This problem is calculated by stability, dynamic stiffness and displacement requirements.

Özet: Deniz petrol yapıların optimizasyonu için son elemanlar üsulu, Reileigh üsulu ve Lagrange katsaylar üsulları kullanılıp. Bu problem yerdeğişme, dinamik Rijitlik ve stabilite sınırı şartlı problemleri çözülüp.

Keywords: dynamic, optimization, stiffness, displacement, offshore.

Introduction

The deep-water offshore platforms of continental shelf are tremendous engineering structures. The height of this platforms reaches to and higher. The weight of structures about 400 000 kN. Therefore, the optimum design of this structure with minimum weight is an actual problem. The block of offshore platform is a space frame construction and is placed under dynamic forces: wind, waves, earthquakes, equipment installed [1]. The structures and design scheme of this platform is in fig.1. In this case the period of natural vibration of the structures becomes co-measurable with the period of external loads. Resonance occurrence is possible. Therefore, dynamic research of this structures is very necessary. Moreover, the block of offshore platform to carry upper structure with drilling oil-derrick, technological equipment and elements of structures are subjected by longitudinal bend and axis force. The structures may lose general stability.

The problem of optimum design is calculated by Lagrange multipliers method, Reileigh correlation [2].

The problem is: minimum weight

$$W = \sum_{i=1}^{n} \rho A_i L_i \Rightarrow \min \qquad (1)$$

where: ρ -density; A_i – cross-section of i-elements; L_i –length of i-elements.

This problem is calculated by stability, dynamic stiffness and displacement requirements. The way of doing this optimisation problem is very popular and widely used in optimal design of structure [2,3,4,5]

* Department of Civil Engineering, İstanbul Kültür Üniversity Şirinevler 34510 İstanbul-Türkiye

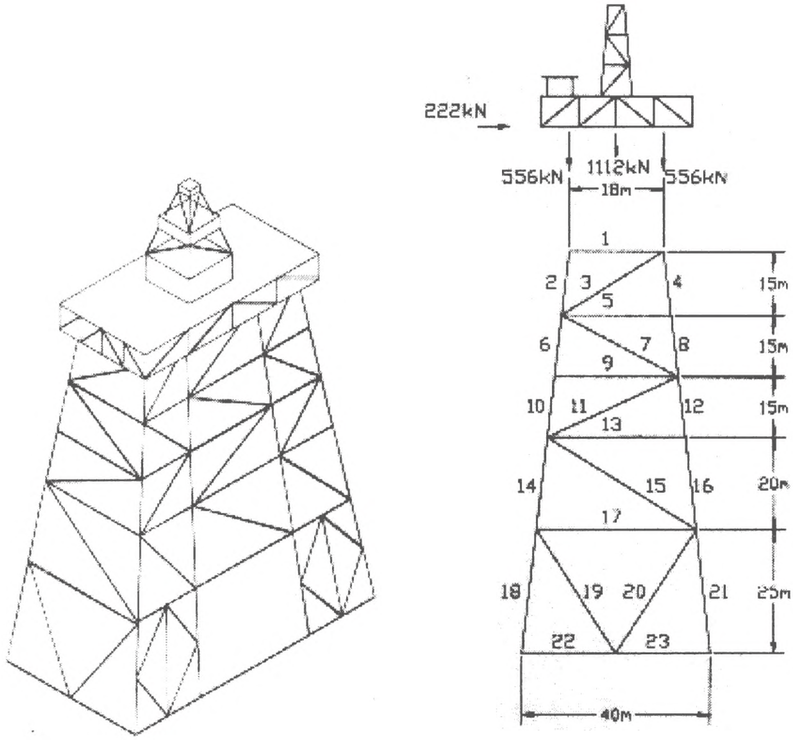

Fig.1 Structure of offshore platform (a) and design scheme (b)

Displacement Requirements

The deep-water offshore platforms are placed under horizontal forces: wind, waves, earthquake, flow, ice e.a. Therefore, displacement requirements are very actually.

The principal requirements are written in form

$$G_i = C_j - \overline{C}_j \qquad (2.1)$$

In this

$$C_i = \{U\}_i^t [K]_i \{S^j\}_i \qquad (2.2)$$

where: $\{U\}_i^t$ -displacement vector with i-element and force; $[K]_i$ -stiffness matrix of i-element.

$$[K]=\frac{EA^2}{L}\begin{bmatrix} \frac{\lambda^2}{A}+\frac{12\mu^2}{L^2} & simmetrik & & & & \\ \left(\frac{1}{A}-\frac{12}{L^2}\right)\mu\lambda & \frac{\mu}{\lambda}+\frac{12\lambda^2}{L^2} & & & & \\ \frac{6\mu}{L} & -\frac{6\lambda}{L} & 4 & & & \\ -\left(\frac{\lambda^2}{A}+\frac{12\mu^2}{L^2}\right) & -\left(\frac{1}{A}-\frac{12}{L^2}\right)\mu\lambda & -\frac{6\mu}{L} & \frac{\lambda^2}{A}+\frac{12\mu^2}{L^2} & & \\ -\left(\frac{1}{A}-\frac{12}{L^2}\right)\mu\lambda & -\left(\frac{\mu^2}{A}+\frac{12\lambda^2}{L^2}\right) & \frac{6\lambda}{L} & \left(\frac{1}{L}-\frac{12}{L^2}\right)\mu\lambda & \frac{\mu^2}{A}+\frac{12\lambda^2}{L^2} & \\ \frac{6\mu}{L} & -\frac{6\lambda}{L} & 2 & -\frac{6\mu}{L} & \frac{6\lambda}{L} & 4 \end{bmatrix}$$

(2.3)

In this λ=cosα μ=sinα
The correlation between forces and displacement are calculated from (2.4)

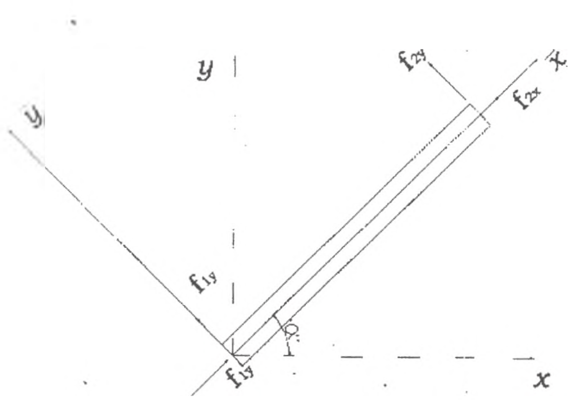

Fig.2 Correlation between forces and displacements

$$\begin{bmatrix} f_{1x} \\ f_{1y} \\ m_1 \\ f_{2x} \\ f_{2y} \\ m_2 \end{bmatrix} = \frac{EA^2}{L} \begin{bmatrix} \frac{1}{A} & & & simmetrik & & \\ 0 & \frac{12}{L^2} & & & & \\ 0 & -\frac{6}{L} & 4 & & & \\ -\frac{1}{A} & 0 & 0 & \frac{1}{A} & & \\ 0 & -\frac{12}{L^2} & \frac{6}{L} & 0 & \frac{12}{L^2} & \\ 0 & -\frac{6}{L} & 2 & 0 & \frac{6}{L} & 4 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \vartheta_2 \end{bmatrix} \tag{2.4}$$

$\{S^j\}_i$ -possible displacement matrix is composed by following principle:

$$K = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \tag{2.5}$$

where K_{ab} reaction in a-joint from displacement b-joint.

Analytical expression for the optimum cross-section of every element is determined using the term of Lagrangian's maximum.

$$L(A,\lambda) = \sum_{i=1}^{n} \rho A_i L_i + \sum_{j=1}^{m} \lambda_i \left(C_j - \overline{C}_j\right) \tag{2.6}$$

where λ_j -Lagrange multipliers.

To get numerical solution of problem, calculation algorithm and computer program were developed.

Stability Requirements

The stability requirements are written down in form

$$G_i = \mu_j - \alpha\mu > 0 \tag{3.1}$$

where μ_j -factic critical force with j-natural mode; $\overline{\mu}$ -lesser critical force; α-coefficient of separate mode.

$$\mu_j = \frac{\eta_j^t K \eta_j}{\eta_j^t K_g \eta_j} \tag{3.2}$$

where η_j –natural vector with j-natural mode; K_g –matrix of geometrical stiffness.

The matrix of geometrical stiffness K_g depends on internal forces by external force P from stiffness.

$$K_g = \frac{\mu}{L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & \frac{6}{5} & \frac{L}{10} & 0 & -\frac{6}{5} & \frac{L}{10} \\ & & \frac{2L^2}{15} & 0 & -\frac{L}{10} & -\frac{L^2}{30} \\ & & & 0 & 0 & 0 \\ & & & & \frac{6}{5} & -\frac{L}{10} \\ & & & & & -\frac{2L^2}{15} \end{bmatrix} \tag{3.3}$$

Dynamics Stiffness Requirements

The dynamics stiffness of structures is calculated by natural frequency. The requirements of frequency is written in form:

$$g = \omega_j^2 - \overline{\omega}^2 \tag{4.1}$$

where $\overline{\omega}$ -minimum frequency of structures.

The value of natural frequency is calculated by Reileigh method

$$\omega_j^2 = \frac{\{\psi\}^t[K]\{\psi\}}{\{\psi\}^t[M]\{\psi\}} \tag{4.2}$$

where $\{\psi\}$-natural vibration mode of structure; [K]-matrix of stiffness system; [M]-matrix of mass with added water mass. The matrix of mass with added water mass is calculated by Reileigh discrete variation method [6].

The analysis of different calculation algorithm of optimization of structures– SAMSEF, PROSSS, TRUSSORT, SPAR, ACCESS end etc. In thair study, C.Fleury, J. Sobiesrczanski-Sobieski, E.Haug, L.Schmit [2,3,4,5] may come to a conclusion that the principles of all programs are finite elements method including following iteration steps:

- fixing the step for modification of cross-section area;
- composition of stiffness matrix (geometrical stiffness and matrix mass with added water mass);
- white down requirements;
- white down displacement (vibration mode, stability mode);
- white down Lagrangian for cross-section A_{j+1} and as compored with Lagrangian for cross-section A_j;
- fixing optimum cross-section which is corresponding maximum Lagrangian.

Conclusion

The calculated result of optimum section and weight of elements by displacement, stability and dynamics stiffness are requirements on table.1.

Table 1

N elem.	L m	A m^2	W kN	Displacement A_0, m^2	W_0, kN	Stability A_0, m^2	W_0, kN	Dynamic A_0, m^2	W_0, kN
1	18	0.645	905.6	0.645	905.6	0.02	28.08	0.645	905.6
2	14.9	1.138	1322.5	0.75	871.6	1.25	1452.7	0.4	464.9
3	27	0.392	825.5	0.2	421.2	0.28	589.7	0.2	421.2
4	14.9	1.138	1322.5	0.75	871.6	1.25	1452.7	0.4	464.9
5	21	0.645	1056.5	0.645	1056.5	0.02	32.76	0.645	749.6
6	14.9	1.138	1322.5	0.85	981.2	1.25	1452.7	0.51	592.7
7	30.8	0.392	941.7	0.25	600.6	0.28	672.6	0.25	600.6
8	14.9	1.138	1322.5	0.85	981.2	1.25	1452.7	0.51	592.7
9	25	0.645	1257.7	0.645	1257.8	0.02	23.2	0.645	1257.7
10	14.9	1.138	1322.5	0.95	1104.1	1.5	1743.3	0.65	755
11	34.4	0.392	1051.8	0.3	805.0	0.3	804.9	0.35	939
12	14.9	1.138	1322.5	0.95	1104.1	1.5	1743.3	0.65	755
13	29	0.645	1459	0.645	1460	0.02	45.24	0.645	1459
14	19.9	1.138	1766.4	1.05	1630	1.8	2794	0.65	1009
15	37.3	0.392	1140.5	0.35	1018.3	0.28	814.6	0.35	1018
16	19.9	1.138	1766.4	1.05	1630	1.8	2794	0.65	1009
17	34	0.645	1710.5	0.645	1710.5	0.02	53.04	0.645	1710
18	24.8	1.138	2201.3	1.15	2224.6	2.2	4255.6	0.65	1257
19	30.2	0.392	923.4	0.35	824.5	0.28	659.5	0.35	824.4
20	30.2	0.392	923.4	0.35	824.5	0.28	659.5	0.35	824.4
21	24.8	1.138	2201.3	1.15	2224.6	2.2	4255.6	0.65	1257
22	20	0.645	1006	0.645	1006	0.22	343.2	0.645	1006.2
23	20	0.645	1006	0.645	1006	0.22	343.2	0.645	1006.2
			30078		26519.5		28466.12		20879

The analysis of result may notice that in case stability requirements cross-section of horizontal elements is almost equal to zero. Then we can remove it and so change topological scheme of structures. That is its may throw aside and all topological scheme of structures change. In this variant, cross-section of vertical elements is very large and therefore, its case are no profitable.

The most profitable are case of dynamics stiffness requirement.

Economical effect with difference of weight for one panel is calculated:

Displacement Requirements

$$W = \frac{W_0 - W}{W_0} 100\% = \frac{30078 - 26519.6}{30078} 100\% = 12\%$$

Stability Requirements

$$W = \frac{W_0 - W}{W_0} 100\% = \frac{30078 - 28466.12}{30078} 100\% = 5.3\%$$

Dynamics Stiffness requirements·

$$W = \frac{W_0 - W}{W_0} 100\% = \frac{30078 - 20879}{30078} 100\% = 30.6\%$$

References

[1] Dawson,T.H. (1983),"*Offshore Structural Engineering*", Englewood cliffs,USA.

[2] Haug E.J. and Arora J.S.,(1979), "*Applied optimal design – mechanical and structural system*". John Wiley and Sons, Inc.New York.

[3] Khot N.S., Sander G.,(1984), "Optimization of structures by the optimality criterion method", *New Direction in optimum structural design*, John Wiley and Sons, Ltd.New York.

[4] Fleury C., Sander G.,(1983), "Methods for Optizing Finite Element Flexual System". *Comp.Meth.Appl.Engrg*.,v.20,No 1,17-38.

[5] "New Direction in Optimum Structural Design",(1984) ed.by. E.Atrek, R.H.Gallagher, K.M.Ragsdell, O.C.Zienkiewicz. John Wiley and Sons, Chichester, New York, Brisbane, Toronto, Singapore.

[6] Mastanzade N.,(1995), "Dynamic behaviour of offshore gravity platform", *Asian Journal of Structural Engineering,* Vol.1,N.1,Summer, Tehran.Iran.