

T.C.
İSTANBUL KÜLTÜR UNIVERSITY
INSTITUTE OF GRADUATE STUDIES

CRASHING IMPLEMENTATION IN CONSTRUCTION
PROJECT CASE

MASTER OF SCIENCE THESIS

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Department: Industrial Engineering

Program: Engineering Management

Supervisor: Assist. Prof. Zeynep GERGİN

JUNE 2022

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M.Sc. Thesis

By Enes ARAP

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I dedicate this humble study;

To the one who was and still is in my soul, the further I go away. My mother Wisal NASER unfortunately who couldn't complete the journey with me (R.I.P).

To my guardian angel and my childhood hero, my father YAHIA ARAP may God grant him a long life, the one who gave me the opportunity of understanding what it is like to be unconditionally loved.

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LIST OF SYMBOLS

CPM	: Critical Path Method.
PERT	: The Program Evaluation and Review Technique.
TCTP	: Time-cost trade-off problem
AON	: Activity on Node
AOA	: Activity on Arrow



Üniversite	: İstanbul Kültür Üniversitesi
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ÖZET

BİR İNŞAAT PROJESİNİN ZAMAN-MALİYET KARŞILAŞMA ANALİZİ

Enes ARAP

İnşaat sektöründe, karar vermede rol oynayan alternatiflerin çeşitliliği, belirsizlik ve hata yapma maliyetleri nedeniyle karar vermek bugün her zamankinden karışık ve geçmişte olduğundan çok daha zorlayıcı bir hale gelmiştir. Bu sebeple deneme yanılma stratejisine veya deneysel bir karara güvenmek son derece kötü bir seçim olmaktadır. İnşaat projelerinde, inşaat alanındaki sürekli değişen koşullar göz önüne alındığında, gider maliyetlerini mümkün olduğunca düşük tutmak için projeyi belirli bir zaman diliminde tamamlamak gereklidir. Bunun yapılmaması, toplam maliyetleri artırır. Bu nedenle, bir zaman-maliyet uzlaşması sağlanmalıdır.

Bu çalışma hedeflenen tamamlanma zamanına ulaşmak için toplam proje süresini en düşük maliyetle sağlamak için bir vaka çalışması sunmaktadır. Projenin şebeke diyagramı kullanılarak Kritik Yol Metodu ile manuel olarak kısaltılması ve matematiksel model geliştirerek Excel yazılımı ile çözülmesi olmak üzere iki yaklaşım karşılaştırılmıştır. Manuel çözüm, proje maliyetini yaklaşık %11,2 oranında artırmıştır ve proje süresini toplam sürenin %35'i kadar olan 115 gün azaltmıştır. Diğer yandan, matematiksel model için Excel çözücü ile bulunan en iyi çözümde proje süresinde aynı azalma proje maliyetinde yaklaşık %9,7'lik bir artışla sağlanmıştır.

Anahtar Kelimeler: Proje Yönetimi, Zaman-maliyet uzlaşması, proje kısaltma.

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ABSTRACT

A TIME-COST TRADEOFF ANALYSIS OF A CONSTRUCTION PROJECT

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Due to the diversity of decision-making alternatives, uncertainty, and cost of making errors, it has become more complex today than ever before, and decision-making has grown considerably more challenging in the construction sector and its atmosphere (Haque, 2010). As a result, relying on a trial-and-error strategy or an experimental decision has become extremely challenging or even a bad choice in decision-making. When dealing with construction projects, it is always necessary to complete the project within a given timeframe to keep expense costs as low as conceivable assuming the continually changing conditions in the construction field. Failure to do so increases the overall costs. Thus, there must be a time-cost trade-off.

This study mainly provides a case study for reducing total project time to reach the completion objective at the least total cost. Two approaches are compared such as crashing the project with the Critical Path Method using the manual network diagramming approach and crashing the project by developing the mathematical model and solving it using Excel software. The manual crashing increased the project cost by almost 11.2% and decreased the project duration by 115 days which is more than 35% of the total duration. The optimum solution provided by the solver for the mathematical model, on the other hand, resulted in a lower cost increase of 9.7% in project costs with the same reduction in total project duration.

Keywords: Project Management, Time-cost tradeoff, Project crashing

1 INTRODUCTION

The rapid expansion in the quantity and scale of projects around the globe over the last decades has resulted in the creation of several issues and administrative aberrations in project management. As a consequence of the absence of effective identification and arrangement of priorities, there was some uncertainty in the visions of organizations and enterprises in managing these projects; this also resulted in the waste of many resources and the wasting of many chances that could have been taken (Putra, Rembulan, & Tannady, 2021).

Essentially, time and money are the two most important factors of a building project, and they are inextricably linked. When a project falls behind schedule, the costs connected with it rise. To fulfill the interests of both the contractor and the customer, the project should be finished on schedule and at the lowest possible cost. So, anytime there is a deviation from the original plan, a schedule crashing technique is used to keep the project on track. The term "project schedule crash" refers to the practice of speeding or decreasing the time of project operations to conclude the project sooner (Kim, Kang, & Hwang, 2012).

Crashing is one of the most critical aspects of completing a building job. Construction planners go through many stages of Construction Project Management to effectively complete the project. As a result, small, big, and complicated construction projects employing Construction Project Management technologies have shown to be effective and have satisfied the needs of the entire customer. To achieve the project deadline, the project manager may need to speed up the project (Islam, Draghici, & Uddin, 2017).

Project managers in the construction sector are constrained by three constraints: the project's timeline, cost, and scope. These three criteria complement one another. Only a project manager who completely understands the triple restriction and knows how to deal with it will be able to complete a successful and high-quality project (Georges; Semaan; Rizk, 2014). the principal benefit of the proposed ACO

over conventional CPM is the capacity of the Ant device to supply precise essential routes and subcritical paths (Nowpada, Rao, & Siresha, 2011). Because project success is assessed by these limits, a project manager must expend additional effort in managing the scope, time, and cost of these three frequently conflicting variables. The factor of schedule is one of the three elements of constraints, is known as time is often a huge risk for a project manager, and it always affects the cost factor, this is because the longer the project takes; the greater the project's cost will be.

In a circumstance where the project management cannot afford to have a project delayed, the project manager will use every option available to them. The most important of them is project collapse. A project crash is a means of reducing the overall duration of a project by allocating greater resources to a certain activity. Whereas project crashing is one of the most important techniques for dealing with project setbacks, it is not an easy process, and it is vital to understand the strategy of the crash and schedule the project. This is because crashing a non-essential activity is worthless since project crashing is only effective on important activities, and the crashing technique is focused on reducing the duration of the critical activities on the path. A project manager should be aware of the tactics for successfully terminating a project (Sahu & Saho, 2014).

To complete the project successfully, managers need to be aware of the challenges they may face. As per the triple restrictions rule, reducing the project's time will increase the project's cost. This is because additional resources are being given to vital operations, allowing the activities to be completed sooner than the intended time frame. The increase of one of the triple restrictions will affect the other constraints. This will have an impact on the overall project's quality. Managing project implementation methodologies is required and vital to integrate project management procedures into the institutions' everyday work culture. Extending the cost and duration of building projects connected to the individual's and society's life results in a loss of quality for these projects (Sahu & Saho, 2014).

This study is implemented to assist project managers with more accurate scheduling crash actions. This hands-on effort will assist in reducing project time at the lowest possible cost and may be used to determine which activities have to be shortened (if the crashed money added from another activity), and what are

activities should be regularly reviewed. It gives a systematic and logical method to decision-making, thus increasing the choice's efficacy. Because the methods specify the start time of the activities, they can be used to track the progress of the project. Furthermore, the sensitivity analysis applied for the modeling approach also provides the upper and lower bounds of the various budget limits within which the solution remains optimal.

It is envisaged that this research will help to achieve the following goals:

1. Show how to utilize project crashing via two approaches such as crashing the project with the Critical Path Method using the manual network modeling approach and crashing the project by developing the mathematical model and solving it using Excel software.

2. Inform the Project Manager of the project case under consideration in the decision-making process and support him in defining the optimal probability duration and cost to meet the project requirements (Islam, Draghici, & Uddin, 2017).

3. Discuss various alternative decisions given the different budget constraints.

The following sections are organized as follows; In the second section Literature Review of the related topic and the basics of the Project, and scheduling are provided. Section 3 explains the methodology. In Section 4 implementation and results are explained. Finally, conclusions and recommendations for future studies are discussed in the last section.

2 LITERATURE REVIEW AND BASICS

2.1 Literature Survey and Previous Studies

Many studies have used CPM and PERT methods and applied them with a manual solution while some others applied mathematical modeling and solved them with some tools such as MATLAB, Lindo, Optimization Model, excel, etc. Also, some studies used both solutions and reported comparative results as this thesis study. The following literature review displayed in Table 1.1 summarizes studies conducted over the previous decade on construction projects.

Table 1.1: Literature review on methods and tools

Reference	Applied Method	Applied Tool
Elmabrouk (2012)	Mathematical Modelling	Linear Programming Using LINDO software
Perez, Elamrousy and Cruz (2018)	Mathematical Modelling	Non-Linear(normally mixed-integer) And Fast-tracking Models
Uğural (2020)	Mathematical Modelling	CPM Crashing Using MATLAB software
Akpan (2012)	Mathematical Modelling	Linear programming methods.
Veludurthi, Anup & Sridevi (2017)	CPM & Mathematical Modelling	CPM & Linear Programming Solver using M.S Excel Solver
Karmarker & Halder (2017)	CPM & Mathematical Modelling	CPM & linear Programming using Lindo software
Sunita, & Snigdha (2013)	Critical Path Method	Crashing by CPM
Najy , AL-Kindi & Mahmoud (2020)	Critical Path method	CPM Crashing, Concurrency Technique, and Crashing

		Techniques (CPCT) Manual Solution
Athira, Andris & Khoiroh (2021)	Critical Path method & Project Evaluation Review Technique	Manual Solution
Çolak, Kesik, Mutman & Keskin (2018)	Project Evaluation Review Technique & Critical Path Method	Manual solution and modeling
Bagshaw (2021)	Critical Path Method	Manual solution
Dawood.Taher & Hamid (2017)	Critical Path method & Project Evaluation Review Technique	Manual solution
Hussein & Al_Khafaji (2020)	Critical Path Method	Simulation in construction project&Linear Programming Method Using Excel
Andiyan, Putra & Rembulan (2021)	Critical Path method & Project Evaluation Review Techique	CPM Manual souldion
Agyei & Holth (2015)	Mathematical Modelling	Linear Programming using LINDO softwere
Uparkar, Chawre & Rehpade (2020)	Critical Path Method (CPM and PERT)	Manual Solution

It is observed that, while scheduling Projects many studies used Mathematical Modeling and some other studies used the Critical Path Method (CPM) or Project Evaluation Review Technique (PERT), and the solutions are supported by various tools such as MATLAB, Lindo, Optimization Model and Excel, etc. Some studies used multiple methods and compared the results.

Elmabrouk (2012) presents contemporary modeling structures to assess extended planning horizon project time-cost crash concerns. He uses linear programming and critical path technique principles to illustrate this system, a simplified representation of a short project. The procedures for resolving these distinct issue formulations were mentioned and solved in the LINDO software. (Elmabrouk, 2012).

Another study suggests unique non-linear models for crashing time and cost Pérez, Elamrousy, and Cruz (2019). This study shows simple terms for scenarios with resources that work in a cooperative and non-collaborative way. The first scenario represents a situation in which all of the resources are provided by a single subcontractor. The duration of the crashed activities, the prices of the crashed activities, and the number of resources involved in the activity compression were all relevant variables in this model. Two of these variables have been mathematically represented as functions of a third variable. On the other hand, the results of our analysis of the two methods were the same for each other (Pérez, Elamrousy, & Cruz, 2018) . The crashing is a generalized type of project plan compression performed by taking steps to reduce the overall length of the project plan, after reviewing a set of substitute products to define how to achieve maximum plan length compression at a cost minimum. (Garg, 2016).

Another study by Uğural (2020) proved to outperform numerous other strategies and a wide range of test issues. In the research, the major goal was to explain, in a simple and accessible manner, how the project's time-cost trade-off assessment decreases the project time, after that it determines the ideal time and money balancing. The critical path technique is used in this study to calculate the time and cost of crashing using the optimization technique. When looking at to study of the researcher and this thesis study, we can say that both have the same goal; reducing the project time in Thanasis of time and the cost of the project. He used MATLAB software, whereas this study uses Excel. (Uğural, 2020).

Akpan (2012) tried a new strategy that compares the accident cost determined at every moment in time with the indirect cost. Since estimating the individual crash cost at any time between the range of normal duration and saturation point is

difficult then the difference between the overall costs of two neighboring locations allows for this. If the direct cost is much more than the direct cost, the lower location is chosen; if the cost is greater, a selection in the other direction is made. Between these two positions, a point of convergence will occur, with the one with the lowest cost being the best option. An extension is also granted in terms of the penalty fee. Both his study and this thesis study used the linear models for crashing (Akpan, 2012).

Veludurthi, Anup, & Sridevi, (2017) demonstrated that the time-cost relationship may be attained utilizing two alternative ways, such as reducing a critical route and redressing the CPM technique's flaw, and it works well for tiny networks with few nodes. The LP Method can be solved using a lot of software programs. His study and our thesis both had the same purpose and employed the same method of solution (linear solution). The aim of the study was how the time-cost analysis of the project reduces project time in the researcher study used to attain hands-on critical path method using Excel solver and Linear Program Solver was utilized to solve the linear programming approach. In any network, the LP approach is superior to CPM since it requires less computing time to give results. (Veludurthi, Anup, & Sridevi, 2017).

Karmarker and Halder (2017) a model is constructed by funding a logical trade-off between time and cost to accelerate the implementation process. To estimate the project's most extended length and price, the critical path method (CPM) was employed, and then the time-cost trade-off problem issue (TCTP) was then represented as a linear programming model. The LINDO software was used to determine the model's solution. the necessary data was acquired through survey and interview discussions with project managers at Chowdhury Construction Company in Dhaka, Bangladesh. Also, this thesis study followed the same path to determine the optimal solution for a time-cost trade-off and employed the same method of resolution, which included mathematical modeling and manual solutions. problem (Karmaker & Halder, 2017).

Sunita and Snigdha (2013) recommended an approach in construction management that is the optimum way for purchasing materials and building the road under the given limitations. It exploited crashes to shorten the project's completion

time even further. The proposed solution has a substantially shorter completion time than the project's real duration. To that aim, it employs crashes not just to save time and money. The projected timetable gave a substantially shorter completion time. CPM planning for road-building projects is a more profitable and viable solution than the actual time utilized by the project, opening the door for its adoption (Sunita & Snigdha, 2013).

Another methodology implemented by Najy, Al-Kindi, and Mhmoud (2020) tried to achieve multiple goals. The paper's method combines concurrency The Multi-threading Segmentation and Crashing Method incorporates partition and crashing methods Both his and our investigations had the same goal and used the same method (manual solution), which included CPM, crashing, and a simultaneous system with partitioning, as well as a new mixed method. The results of the researcher's study showed that the proposed hybrid strategy (CPCT) produced more acceptable outcomes than the other strategies, with the duration of the project and cost decreasing by 19.5 percent and 2.6 percent, respectively. In comparison, the total number of workers increased by 8.8 percent (Najy, AL_Kindi, & Mahmoud, 2020).

In another paper by Athira, Andris, and Khoiroh, (2021) Project renovation was used since the project's implementation was delayed. Delays arise as a result of a shortage of available staff. This delay will almost probably increase expenditures. Both studies had the same purpose and used the same way (manual solution), the CPM-PERT approach. This research aimed to shorten the time and save expenses while maintaining quality requirements Also, our study used the same road to know the best solution for a trade-off between time and cost and used the same way of a resolution by using (Linear and Manual solutions) with Excel. According to the findings of the researcher's study, it may be expedited for 36 working days for Rp18,932,500, with additional processing charges of Rp70,000 and overtime costs of Rp1,635,000, for a total cost of Rp18,932,500 (Athira, Andris, & Khoiroh, 2021).

As a consequence of the mathematical model by Çolak, Kesik, Mutman, and Keskin, (2018), the project's projected completion time had been predicted to be studied utilizing a project management methodology. Initially, both studies had the same purpose and used the same way (manual solution), the CPM-PERT approach.

But the difference was in the type of project. A project planning in the researcher study approach was used to calculate the completion time and essential tasks. As a consequence of the mathematical model, the project's projected completion time had predicted to be 54.9 days. A crashing model was created to convert the construction project for crucial tasks to avoid delaying the tramline project. This project crashing model aims to calculate the cost necessary to complete the project in 45 days. The study of the researcher's cost of project failure for important tasks had computed using a mathematical programming model of 1659.4 TLs. The result of the study is that it is possible to complete the project ten days early for around 1% of the overall project cost (Çolak, Kesik, Mutman, & Keskin, 2018).

In the study by Bagshaw (2021) an idea of the Project Evaluation Review Technique and used manual solutions (CPM and PERT Method) in project management has been explored. In terms of project time and critical activities, the technique and efficacy of quantitative decision-making techniques in project management were examined, including the use of the project evaluation review technique and the critical route method. While both techniques are effective in achieving project management, the interconnectedness and linking of activities across a project's life cycle are crucial. And both studies concluded that both strategies provide good results (Bagshaw, 2021).

A statement on using non-activated and underutilized scientific planning methods, such as network planning methods, in planning construction projects in the Baghdad governorate to calculate the final duration of the project considering available resources Dawood, Tahir & Hamid (2016) presented the critical route technique to the project, the unified school in Baghdad, draws the network of work according to the verified times of the project activities and calculates the expedited time while presenting numerous possibilities for the time-cost trade-off (Dawood, Taher, & Hamid, 2017).

A simulation is processed in the time scheduling of construction projects, utilizing a real-world project as a case study by Hussein and Al-KKhafaji (2020) demonstrated that there was no one-of-a-kind setting to carry out a building project in terms of length or expense. Instead, there are risk probabilities in these traits that must be considered throughout the planning phase. Simulation is a helpful

technique for anticipating numerous project lengths or cost scenarios since it allows the management to have a broad picture of the construction during the planning phase (Hussein & AL_Kkhafaji, 2020).

Andiyan, Putra, and Rembulan (2021) mention that proper planning is necessary to avoid project delays at one construction business in Indonesia, where the company was dealing with several issues. The Critical Path Method (CPM) and crashing were used to calculate late payments that the corporation could save the researcher. The CPM-Crashing approach could shorten project length by increasing the resource budget, allowing the organization to avoid paying late fees. The researcher generated a new project schedule utilizing CPM-Project Evaluation Review Technic (PERT) and Critical Chain Project Management (CCPM). CPM-PERT provided a likelihood of project completion, whereas CCPM provided a new project schedule. According to the result of this article, the likelihood of a project being delayed is quite low, with just 55% of projects finishing on time (Andiyan, Putra, & Rembulan, 2021).

By making a trade-off between the cost and the shortest possible time to complete the construction project indicated by Agyei and Holth (2015). Angel Estates and Construction Ltd. provided data on the cost and time of the activities involved. The critical path method (CPM) and project evaluation and review technique (PERT) were utilized for the study and used Linear Programming Using LINDO software (Agyei & Holth, 2015).

Another Crashing approach was utilized in this study by Uparkar, Chawre, and Rehpade (2020) to assess the balance between project length and expense In India, The Metro rail project is both a new mass rapid transit system (MRTS) and a multi-industry hub. The study suggests that floats allow better flexibility in the scheduling of non-critical processes using the PERT approach, based on the three techniques presented in the computational section (Uparkar, Chawre, & Rehpade, 2020)

In the study by Abd (2018),. The project was chosen because the work had been delayed, causing a delay in completion at the scheduled period. Both studies had the same goal and employed the same method (manual solution) by employing the critical path method (CPM), a project scheduling and management technique. For the Time-Cost Tradeoff Problem, construct the project, assess the links between

jobs, compute slack and crucial periods, and decide the most extended length and cost to complete the project. The discrete time-cost tradeoff issue DTCTP was subsequently turned into an optimization programming language OPL model using the mixed-integer programming paradigm. In this scenario, the IBM ILOG CPLEX Optimizing Studio by another researcher was used to discover (ABD, 2018).

The paper by Ashok (2011) proposed a simple way of providing an efficient solution to the problem of selecting activity modes for project crashes. The goal was how the time and cost trade-off of the project study reduces project time. The critical path technique (CPM) is utilized to calculate the time and cost of crashing. When looking at our study used the same road of a linear solution, but in a different program, it used linear programming using Excel. (CPM) the method demonstrated by the researcher used; In all experiments that yielded good and adequate outcomes, the method operated effectively in practice on a wide range of time-cost crashing situations (Ashok, Biswajit, & Jibit, 2011).

2.2 Project Scheduling and Crashing Basics

Project scheduling is calculating the start, finish, and completion time of the activities with precedence relations. A scheduling model and a tool are used to utilize the project information in a schedule as displayed in Figure 2.1. In this chapter, general information on the project scheduling by critical path method (CPM) is introduced and the basic steps of this approach for different types of networks are explained. Also, when a project falls behind schedule it may be necessary to crash the tasks to ensure that the project is completed on time. Project crashing is applied to reduce the amount of time that takes to complete the project. The following sections explain the principles of network diagramming, the CPM method, and crashing.

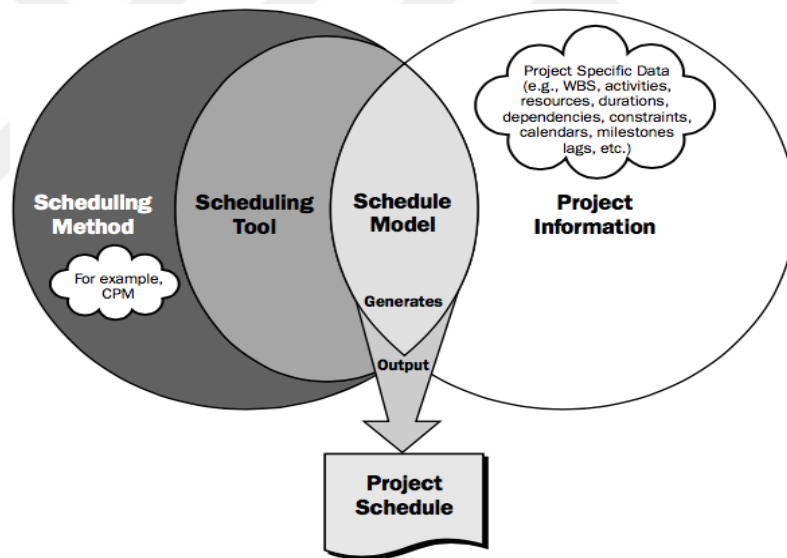


Figure 2. 1: Project Schedule Development

2.2.1 Network Diagramming

The network diagram of a project shows the paths of the project. Activities can be displayed on nodes or arrows. In activity-on-node (AON) approach, activities are shown with cells called a node, and the relationships of interdependence between activities are shown by the arrows whereas activities are denoted on arrows in Activity on Arrow diagrams. Two networks for the same project are presented in Figures 2.2 and 2.3.

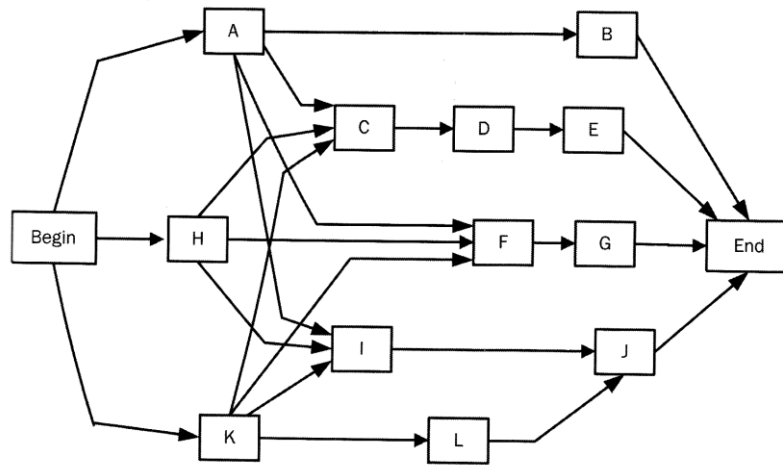


Figure 2. 2: Activity on Node (AON) Network Diagram

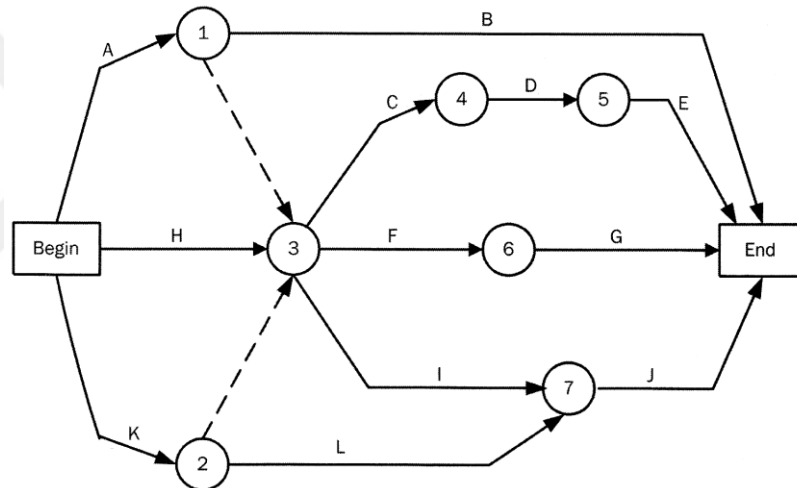


Figure 2. 3: Activity On Arrow (AOA) Network Diagram

Dependence is essentially a connection that occurs between two actions that are performed in tandem. In this case, the term "dependent on activity (A)" refers to the fact that activity (B) provides a deliverable that is required to complete the activity's work (A). There are four kinds of activity dependencies to consider. (Tmrakar, 2013)

- **Finish to Start (FS) dependency:** When two actions are connected by a finish-to-start dependency, it signifies that the activity predecessor have be finished before the beginning of the successor. Finish-to-start dependencies are general in software development predecessor activity's right edge and

reaching the successor activity's left edge., it becomes clear to see the process of finish-to-start dependence (Güneri & Mazlum, 2015) This type of dependency relationship is shown in Figure 2.4.

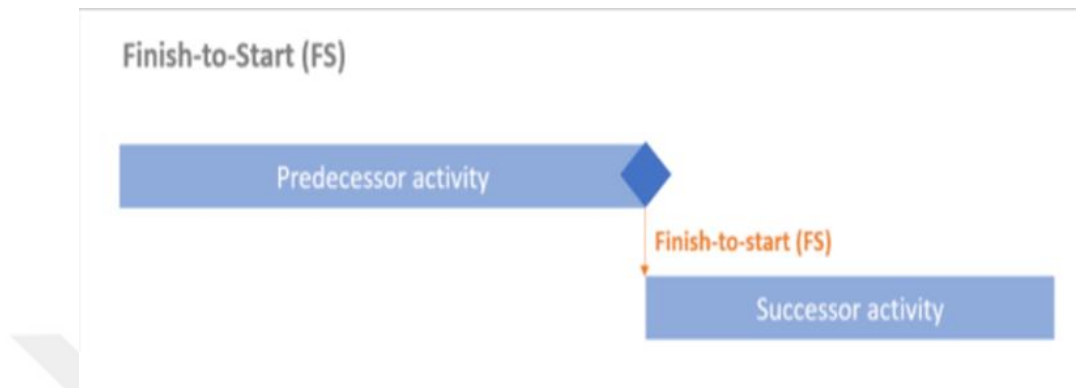


Figure 2. 4: Finish-to-Start precedence relationship

- **Start to Start (SS) dependency:** When two activities are linked by a start-to-start dependency, it signifies that the predecessor action must have completed its execution before the successor activity may commence its execution. As seen in the diagram, the start-to-start dependency is represented by an arrow that starts at the left edge of the predecessor (A) and extends to the left edge of the successor (B). Figure 2.5.

Start-to-Start (SS)

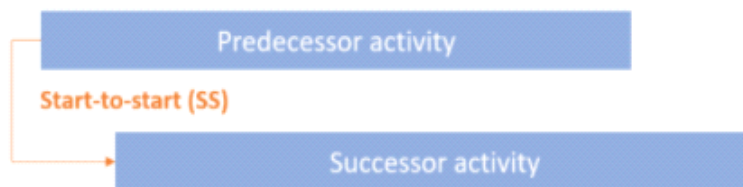


Figure 2. 5: Start-to-Start precedence relationship

- **Start to Finish (SF) dependency:** This logical connection necessitates the completion of a predecessor action before the completion of a successor activity. The finish-to-finish (SF) criteria are a little more complex than the

finish-to-start (FS) and start-to-start (SS) criteria. Action (B) cannot be performed any sooner than activity (A) was started in this situation. The start-to-finish relationship between two activities is represented by starting at the left edge of activity (A) and progressing to the right edge of activity (B) (Aliyu, A. M., 2012). This type of dependency relationship is shown in Figure 2.6.

Start-to-Finish (SF)

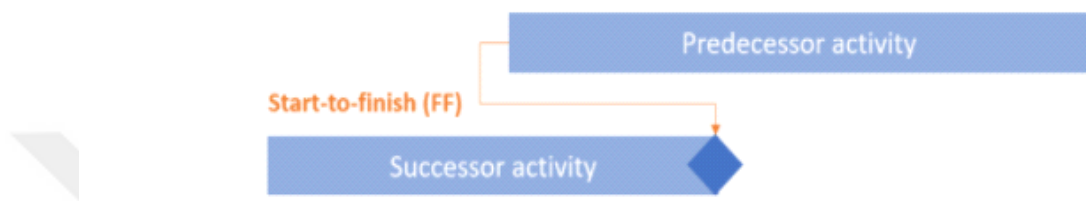


Figure 2. 6: Start-to-Finish precedence relationship

- **Finish to Finish (FF) dependency:** In this form of a logical link, a successor activity is dependent on Before it can be done, the preceding task must be completed. This form of dependency may also occur in conjunction with the start-to-start linkages that occur between the same types of activities. Specifically, the finish-to-finish (FF) dependence specifies that action (B) cannot be completed earlier than activity (A). It is shown as an arrow extending from the right edge of activity (A) to the right edge of activity (B) (Aliyu, A. M., 2012) as seen in Figure 2.7.

Finish-to-Finish (FF)



Figure 2. 7: Finish-to-Finish precedence relationship

2.2.2 Critical Path Method (CPM)

The critical path method (CPM) is a fundamental numerical project management technique. that was developed in the 1960s. CPM calculates the lowest amount of time required to finish the Project by assuming predictable activity completion timings and using the Project's network diagram. (Tiwari, Sajay; Johari, Sparsh, 2015) CPM presupposes that each action has a fixed time estimate and that activity times are not variable. The learning technique is primarily based totally on the precept of the usage of problems methodology, which shows higher know-how of the conceptual element and a more capacity for growing CPM diagrams (Gonzalez & Opazo-Vega, 2014). The output of the method gives a detailed network diagram showing the following for the project:

- Early start of Activities (ES)
- Early finish of Activities (EF)
- Late start of the activities (LS)
- Late finish of the activities (LF)
- Slack (float) times of the activities (S)
- Critical activities (Slack=09 and critical path(s))

2.2.3 Project Crashing

In many projects, there is a kind of relationship between the cost and the length of the project that must be considered. When a project falls behind schedule, it may be necessary to allocate more personnel to the task to expedite the process. Crashing is a strategy for analyzing cost and schedule tradeoffs to find how to get the most compression for the least amount of the cost. In other words, Crashing is the procedure of fine-tuning your project schedule to shorten the time it takes to accomplish it. When stakeholders want an early delivery but don't want to limit the scope of the assignment, this is a potential choice. Crashing is often done in the course of project management if it is possible to compress the time of any of the activities at an extra expense to reduce the overall project duration (Panth & Bidar, 2017). Furthermore, most projects have a due date (or, in some cases, a "soft deadline") for completion, which forces a project to fail.

In the case of a project that is completed on schedule, there may be a possibility to crash the project by employing extra work resources or acquiring additional equipment (Adate, Goel, & Sundaramali, 2017). Managers must weigh the costs and advantages of moving the project forward more quickly.

Project managers these days rely on planning tools based on project networking techniques such as the critical path method (CPM) to determine overall project duration and activity transition time. (Tiwari, Sajay; Johari, Sparsh, 2015)

The following steps are applied in crashing implementations (Geda, Mohammed Woyeso, 2014)

- The cost-effectiveness of crashing actions on the essential pathways is identified (To have an effect, it may be required to crash multiple activities). The game is over if no series of crashes result in a net gain.
- Using the most cost-effective crash, implementation is continued as long as, It is cost-effective until the involved pathways are no longer necessary.
- A crash in step 2 may result in the creation of new critical routes in the next stage. Any more critical paths are identified by updating the network. If this is the case, the process is restarted in step 1.

Different methods are adopted for crashing implementations varying from manually applied to mathematical programming. The following section gives some studies on Project Crashing implementations in construction projects.

3 METHODOLOGY

The methodology used to conduct this study as displayed in Figure 3.1 is as follows; Initially, data on the project activities are collected. The network diagram is developed and CPM is applied to calculate the project's critical path. Manual crashing is applied with the number of required iterations for calculating the lowest cost for a maximum amount of crashing. Then the mathematical model of the problem is developed and solved with the Excel solver to find the strategical crushing solution for the project. Lastly, The outcomes of the two methods (manual and LP optimization) are compared.

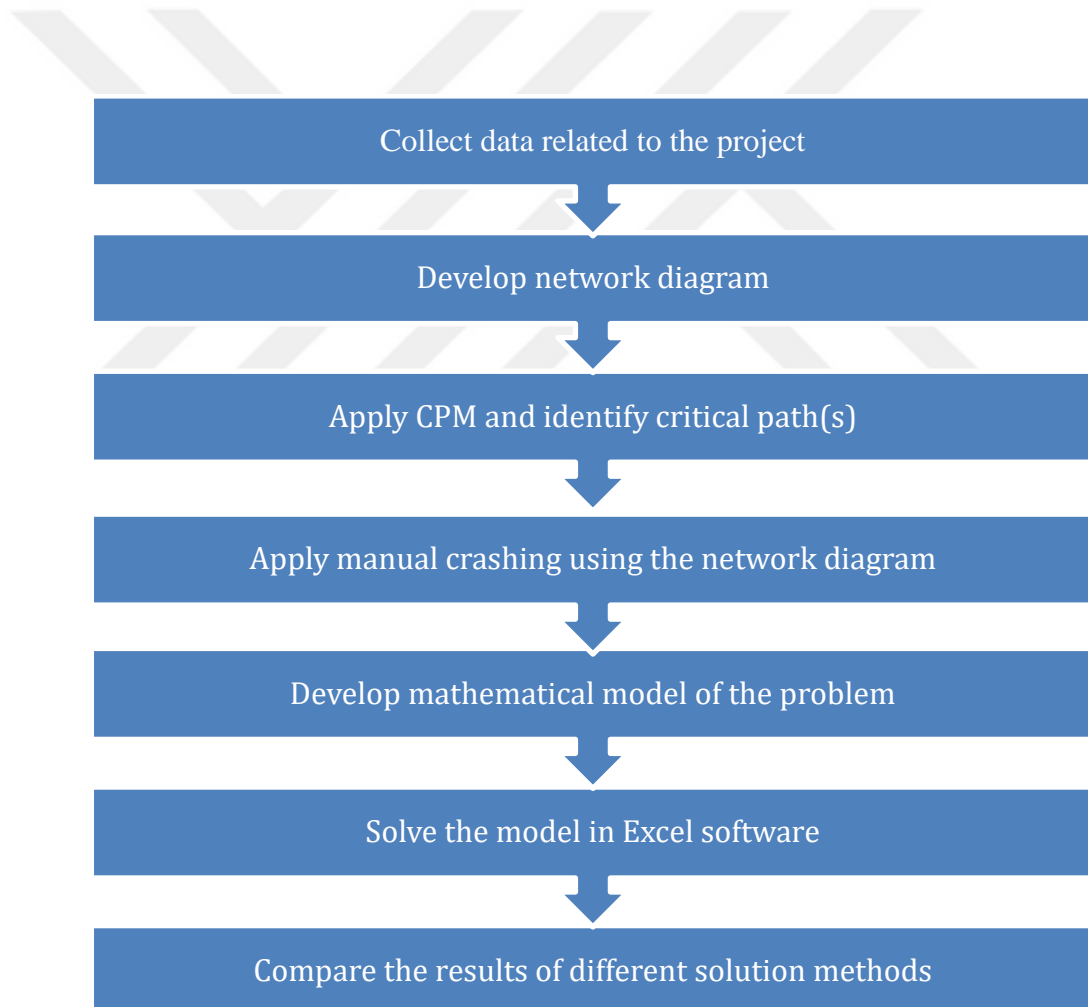


Figure 3. 1: Methodology flow of the study

3.1 Data Collection

Data collected for the project are as follows:

1. Activities of the project
2. Predecessor relationships of activities
3. Activity normal durations
4. Activity crashed durations
5. (The budget limit for crashing)
6. (Target duration of the project)

3.2 Network diagramming and CPM

A network is a graph that depicts each activity that will be performed, as well as its predecessors and successors. Activity on Node (AON) network diagramming is used to develop the network diagram of the project to apply the CPM method (Zareei, 2018). Predecessor relations and the normal activity durations are employed to estimate the critical path(s) of the project. The nodes are distributed to cells to enter the outputs of the computations for Early Start (ES), Early Finish (EF), Late Start (LS), and Late Finish (LF) times of activities as shown in Figure 3.2.

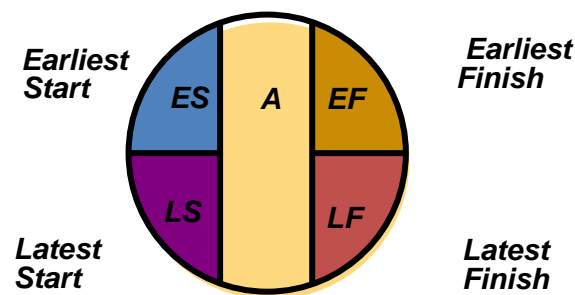


Figure 3. 2: Cells of a node in the network diagram

Below are the steps of applying the CPM method manually.

1. Construct the AON network connecting all of the activities
2. Appoint durations (t_i) for each activity

3. Calculate the Early Start (ES) and Early Finish (EF) times of each activity by using the forward pass.

$$EF_i = ES_i + t_i$$

4. The early start of the succeeding activity is equal to the EF of the preceding activity. If there are more than 2 preceding activities (i.e., j,k,m) before activity I then the maximum EF of the preceding activities is chosen as the ES of the succeeding activity.

$$ES_i = \max (EF_j, EF_k, EF_m)$$

5. calculating the Late Start (LS) and Late Finish (LF) times of each activity by using the backward pass.

$$LS_i = LF_i - t_i$$

6. Late Finish of the preceding activity is equal to the LS of the preceding activity. If there are more than 2 succeeding activities (i.e., j,k,m) after activity I then the minimum LS of the succeeding activities is chosen as the LF of the preceding activity.

$$LF_i = \min (LS_j, LS_k, LS_m)$$

7. Identify the slack times

$$S_i = EF_i - ES_i \text{ or } S_i = LF_i - LS_i$$

8. If Slack time is zero (0) that indicates the critical activity; Combine the critic
9. Calculate the critical path, which is the longest time path through the network. all activities to identify the critical path(s)

3.3 Manual Crashing

Crashing is the next stage is to find the critical activity on a path with the least crash cost per time and crash that activities in rounds paying attention to the calculations on the critical path. To meet the timeframe and our time or cost target.

Manual crashing is applied using the network diagram provided by the CPM method. The following steps are applied.

1. Estimate the crash cost per period. the following formula may be applied to estimate the cost-time slope If crash costs are linear over time;

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

1. If there is only one critical path, choose the critical path activity that (a) can still be crashed and (b) has the lowest crash cost per period (cost slope).
2. If there are several critical paths, choose one activity from each critical path so that (a) each picked activity can still crash and (b) the overall crash cost of all selected activities is the lowest. It's important to note that a single activity may be shared by more than one critical path.
3. Control the output duration by applying forward and backward passes to all activity times.
4. When the due date has passed, stop if it is not possible. Return to Step 2

There will be no end to the crash as long as the lowest cost time slope of the new route is taken into account in the new critical path's crash calculation (Adebowale, S. A.; Oluboyede, E. D., 2011).

3.4 Mathematical Model of the Crashing Problem

In this section, the mathematical representation of the crashing problem is described.

Indices:

i : activity index , $i = 1, 2, \dots, N + 1$

$N+1$ is the activity indicating the finish of the project

Parameters:

t_{iN} : Normal duration of activity i ,

t_{iC} : Crashed duration of activity i ,

C_{iN} : Normal cost of activity i ,
 C_{iC} : Crashed cost of activity i ,
 C_i : Cost slope (crash cost per period) for activity
 TD : Target Duration of the Project

Decision Variables:

Y_i : crashed duration of activity i (number of periods it is crashed)
 X_i : the early finish time for activity i

Mathematical Model:

Objective function.

$$\text{Min } Z = \sum C_i y_i \quad (3.0) \quad i=1, \dots, z$$

Subject to.

$$Y_i \leq t_{iN} - t_{iC} \quad (3.1)$$

$$X_i - X_{i-1} + Y_i \geq t_{iN} \quad (3.2)$$

$$X_i \geq 0 \quad (3.3)$$

$$Y_i \geq 0 \quad (3.4)$$

$$X_1 = 0 \quad (3.5)$$

$$X_z \leq TD \quad (3.6)$$

The objective function in Equation (3.0) is for the total crashing cost of the project to be minimized. Constraint (3.1) ensures that the number of periods each activity is crashed does not exceed its maximum possible value. Constraint (3.2) guarantees, for each activity i and each of its immediate predecessors, that the early completion time for its predecessor plus its own normal completion time minus the number of periods it is crashed. Constraints (3.3) and (3.4) are nonnegativity restrictions on the decision variables. Because Event 1 will start at the start of the

project, we assign the occurrence time for the first node to zero in constraint (3.5). Finally, a target duration (TD) is provided for the project and entered as the early finish of the last node in Constraint (3.6.)



4 IMPLEMENTATION AND RESULTS

4.1 Project Data

4.1.1 Project Information

The project selected for this study is a construction site in Damascus, Syria. The project is a construction project (a cooling tower). The research project under construction was supposed to start in the 2nd week of October 2007 to be accomplished in the 2nd week of April 2010. The contract's starting cost was 153590 million \$. The project was abandoned. It was later revoked, and. The project has not yet been completed at the time of this research. Because of the war and the rapid currency exchange, crashing is required to achieve a balance between the duration of the project and its additional costs after the crashing process to earn a profit on the project despite the currency exchange effect.

4.1.2 Project Problem

Time and money are the two most important factors in construction projects, and they are inextricably linked. When a project falls behind schedule, the costs connected with it rise. Also, we recently have another important factor in construction operation, which is the recent extreme changes in the exchange rates that impact negatively on the construction business. This rapid change in currency exchange rates which cannot be prognosis before, due to the economic wars and the political decisions of the middle-developed countries, affect the costs of construction material. Hence, these fluctuations in the foreign currency rate have a vital effect on the success or failure of the profit targets of construction institutions. As a result, the construction business, the companies whose costs and profits consist of projects, should find a dynamic approach to protect themselves from these rapid cost fluctuations.

As the duration of the project increases, it is more subject to these fluctuations. On the other hand, shortening the duration also requires a certain amount of budget increase. Hence, the objective of this study is to find the best trade-off for this problem.

4.1.3 Project Activities and CPM Method

Data collected for the project are as follows.

1. **Activities of the project:** There are seventeen activities in the program chart. The duration of each activity was calculated on a daily bases. An action was designated by a code, such as (X1 ..., X17.) on Table (4.1) The data was through the company that was responsible for the project, and physical contact was made with the project engineer.
2. **Predecessor relationships of activities:** Below Table (4.1) contains a description of predecessor relationships of each action as well as the links between them
3. **Activity normal durations:** Shown in Table (4.1).
4. **Activity crashed durations:** Shown in the table (4.1).
5. **The budget limit for crashing:** Because of the difficult economic conditions in Syria and the rapid currency exchange of the Syrian pound vs American dollars, the materials that their cost-based on dollars may force the construction projects' profit. So, a balance between the duration of the project and its additional costs after the crushing process to achieve the project profits regarding the effect of the currency exchange is to be considered as an official solution to such situations. Some economical studies, it has investigated that the Syrian pound has lost its value vs American dollar sharply in the last ten years more than 80% of its value. So many economical studies show that the project has to be crashed by more than 35%.
6. **Target duration of the project:** The target that wants to be reached in the project. The target that wants to be reached in the project.
 - Minimize the project duration by more than 35% from the full duration of the project
 - The project cost shouldn't exceed 12 % of the normal cost.

The project activities in Table 4.1 display information on all of the project's work packages (will be called the activities of the schedule), as well as predecessor activities, normal completion time, crash completion time, normal cost, and crash

cost. There are 17 activities to be completed in this project, and the average time it takes to complete them is days.

Table 4. 1: The activities of the project.

Activities Of the studied Project	Activity Description	Predecessor Relationships of Activity	Time activity (Normal time) (Days) t_{iN}	Time Crashing (Crash Time) (Days) t_{iC}
Activity A	Making Steel Structures dismantled	-	46	30
Activity B	Transferring the remnant of the steel	A	15	7
Activity C	Making the concret beam braced	A	30	15
Activity D	Tiing the conctring beams	C	16	7
Activity E	Assembling Pre-cast Matiriales	B	90	60
Activity F	Preparing the Wall Pre-casted	D	80	50
Activity G	Concrete the Columns	D	90	60
Activity H	Concrete the Roof's Beams	F, G	46	30
Activity I	Constructing a room for the elctric	E	15	7
Activity J	Concerting Roof's Culumns	H, I	15	10
Activity K	Combining the peecasted roofs	J	31	7
Activity L	Fabricateing a circuit Outlet	-	45	30
Activity M	Construct Steel Ladder	-	60	30
Activity N	Assemble Hand Rail	-	31	7
Activity O	Install the steel Leader	M	14	7
Activity P	Concrete the circuit Outlet	L	15	7
Activity Q	Set up a Steel Handrail	N, O	15	7

4.2 Network Diagram of the Project and CPM

The AON network diagram of the project is provided below in Figure 4.1.

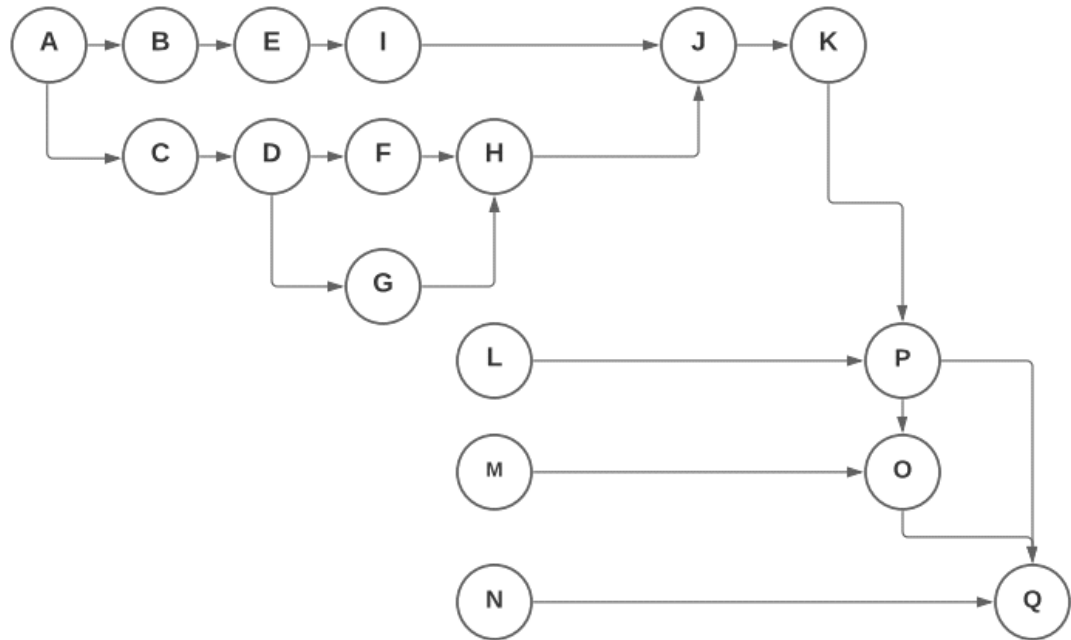


Figure 4. 1: Network Diagram of the project activities.

The steps of applying the CPM method manually (Biswas, Karmaker, & Biswas, 2016);

1. Drawing the AON network connecting all activities from A To Q
 - a. From Figure (3.2)
2. Assign durations (t_i) to each activity
3. Apply forward pass to estimate the Early Start (ES) and Early Finish (EF) times of each activity.
 - a. $EF_i = ES_i + t_i$
4. The early start of the succeeding activity is equal to the EF of the preceding activity. If there are more than 2 preceding activities (i.e., j,k,m) before activity i then the maximum EF of the preceding activities are chosen as the ES of the succeeding activity.
 - a. $ES_i = \max (EF_j, EF_k, EF_m)$
5. Apply backward pass to estimate the Late Start (LS) and Late Finish (LF) times of each individual activity.

- a. $LS_i = LF_i - t_i$
6. Late Finish of the succeeding activity is equal to the LS of the succeeding activity. If there are more than 2 succeeding activities (i.e., j,k,m) after activity i then the minimum LS of the succeeding activities are chosen as the LS of the preceding activity.
 - a. $LS_i = \min(LF_j, LF_k, LF_m)$
7. Identify the slack times
 - a. $S_i = EF_i - ES_i$ or $S_i = LF_i - LS_i$
8. If Slack time is zero (0) that indicates the critical activity; Combine the critical activities to identify the critical path(s)
9. Or assemble the longest time path through the network – this is called the critical path

The outputs of the steps give the critical path. All the detailed calculations on nodes are provided in Table 4.2.

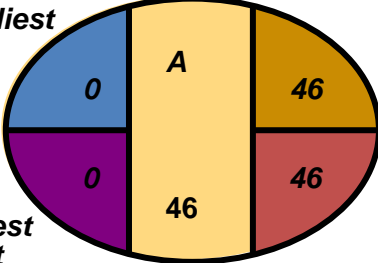
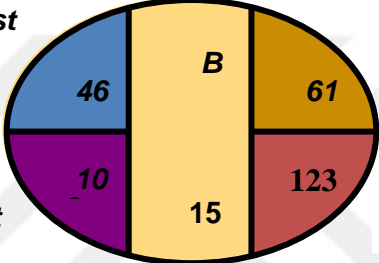
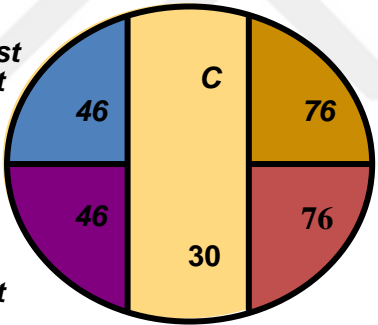
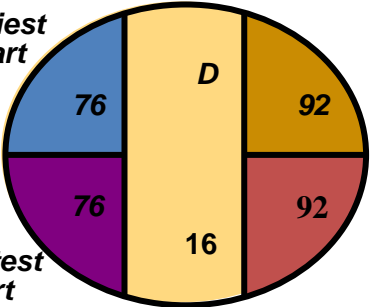
Establishment of Critical Path:

The shortest path to project completion. And the project's paths:

- $A+B+E+I+J+K+P+O+Q = 256$
- $A+B+E+J+K+P+Q = 242$
- $A+C+D+F+H+J+K+P+O+Q = 308$
- $A+C+D+F+H+J+K+P+Q = 294$
- $A+C+D+G+H+J+K+P+Q = 304$
- $A+C+D+G+H+J+K+P+O+Q = 318$
- $L+P+Q = 75$
- $L+P+O+Q = 89$
- $M+O+Q = 90$

Since; A--D--G--H--J--K--P--O--Q is equal to 318 days with the largest value then it is assembled as the critical path

Table 4. 2: CPM implementation on each node of the network

The Activity	AON Network Cell
A activity	
B activity	
C activity	
D activity	

E activity	<p>Earliest Start: 61, Earliest Finish: 151, Latest Start: 12, Latest Finish: 213, Activity: E, Duration: 90</p>
F activity	<p>Earliest Start: 92, Earliest Finish: 172, Latest Start: 102, Latest Finish: 182, Activity: F, Duration: 80</p>
G activity	<p>Earliest Start: 92, Earliest Finish: 182, Latest Start: 92, Latest Finish: 182, Activity: G, Duration: 90</p>
H activity	<p>Earliest Start: 182, Earliest Finish: 228, Latest Start: 182, Latest Finish: 228, Activity: H, Duration: 46</p>
I activity	<p>Earliest Start: 151, Earliest Finish: 166, Latest Start: 213, Latest Finish: 228, Activity: I, Duration: 1</p>

<p>J activity</p>	<p>Earliest Start: 228 Earliest Finish: 243 Latest Start: 228 Latest Finish: 243 Activity: J Duration: 15</p>
<p>K activity</p>	<p>Earliest Start: 243 Earliest Finish: 274 Latest Start: 243 Latest Finish: 274 Activity: K Duration: 31</p>
<p>L activity</p>	<p>Earliest Start: 45 Earliest Finish: 45 Latest Start: 0 Latest Finish: 0 Activity: L Duration: 45</p>
<p>M activity</p>	<p>Earliest Start: 0 Earliest Finish: 60 Latest Start: 0 Latest Finish: 0 Activity: M Duration: 60</p>

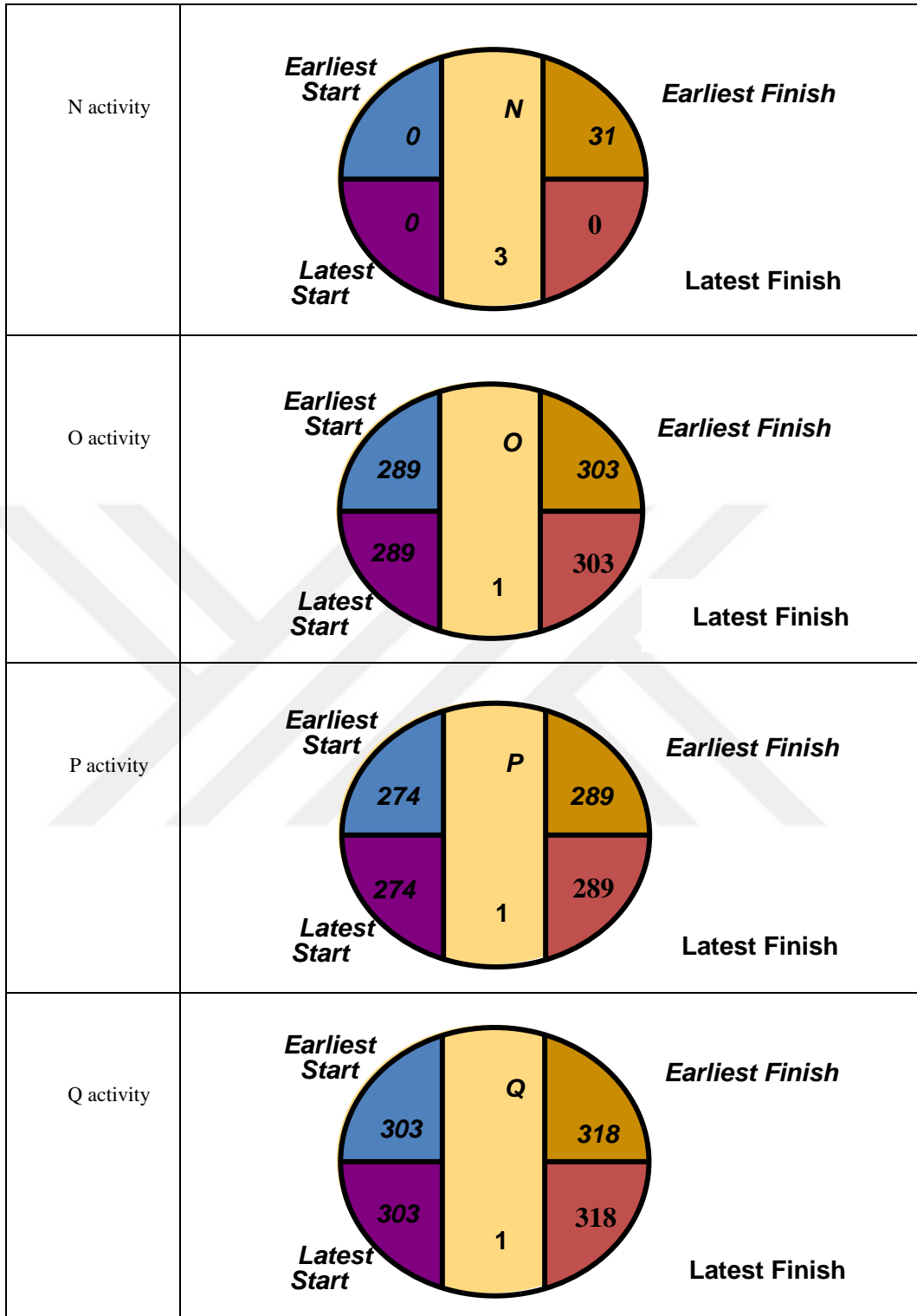


Figure 4.2. and 4.3 gives the ES-EF and LS-LF Gantt Charts of the project based on the calculations.

ES-EF Gantt Chart

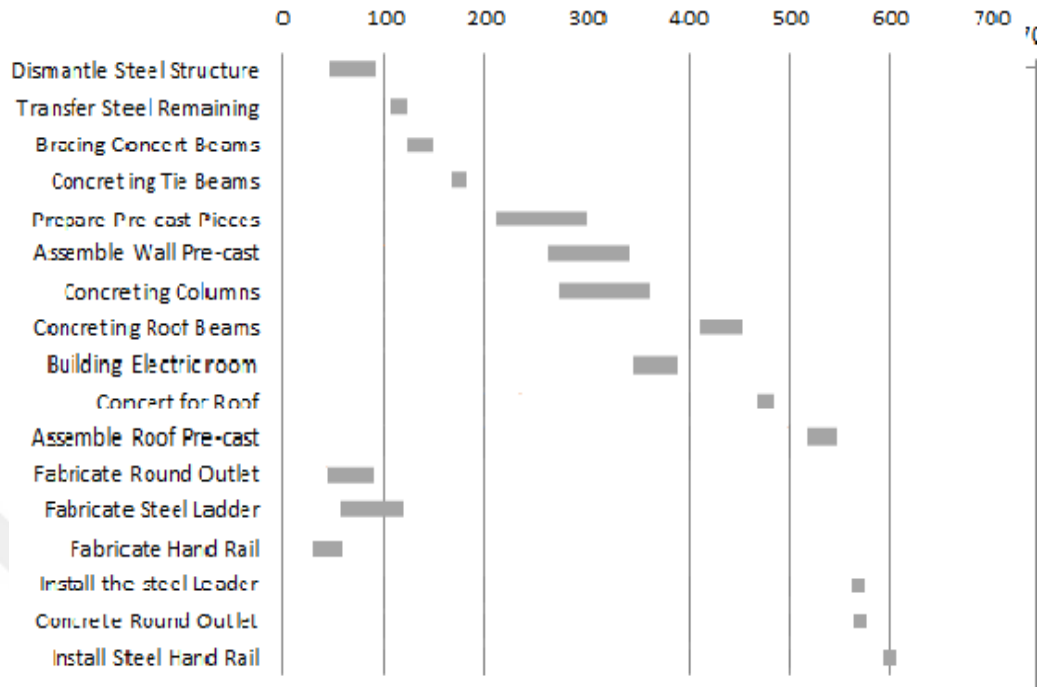


Figure 4. 2: ES-EF Gantt chart.

LS-LF Gantt Chart

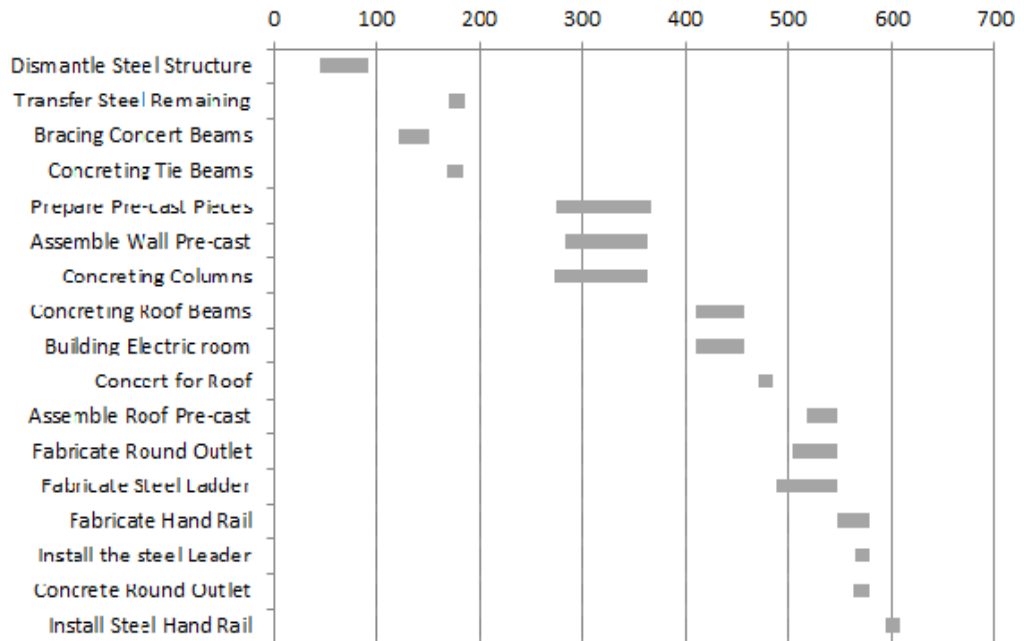


Figure 4. 3: LS-LF Gantt chart

4.3 CPM and Manual Crashing Solution

Normal and crashing costs of material and labor costs are incurred and manual crashing is applied using the network diagram provided by the CPM method. The following steps mentioned in the methodology section are applied with the assumption of the project's normal and crashed costs listed on Table 4.3 include both fixed and variable costs for each activity.

Table 4. 3: Time-Cost information for the project activities.

Activity	Activity Durations (Days)		Activity Costs (\$)		Crash Cost (\$/day)	Presence on the critical path
	Normal (t_{iN})	Crash (t_{iC})	Normal (C_{iN})	Crash (C_{iC})	Cost slope (C_i)	
A	46	30	7650	9500	115.6250	YES
B	15	7	2300	3500	150.0000	NO
C	30	15	3000	4000	66.6660	YES
D	16	7	3500	4500	111.1111	YES
E	90	60	45000	50000	166.6667	NO
F	80	50	20500	25000	150	NO
G	90	60	30000	37000	233.3333	YES
H	46	30	11500	15000	218.75	YES
I	15	7	2000	7500	687.5	NO
J	15	10	6000	8000	400	YES
K	31	7	3200	5500	95.8333	YES
L	45	30	6000	7000	66.6667	NO
M	60	30	5300	7500	73.3333	NO
N	31	7	1000	2000	41.6667	NO
O	14	7	2200	3000	114.2857	YES
P	15	7	4000	5000	125	YES
Q	15	7	440	800	45	YES
Total normal cost:			153,590\$			

Crashing iterations:

The main iteration:

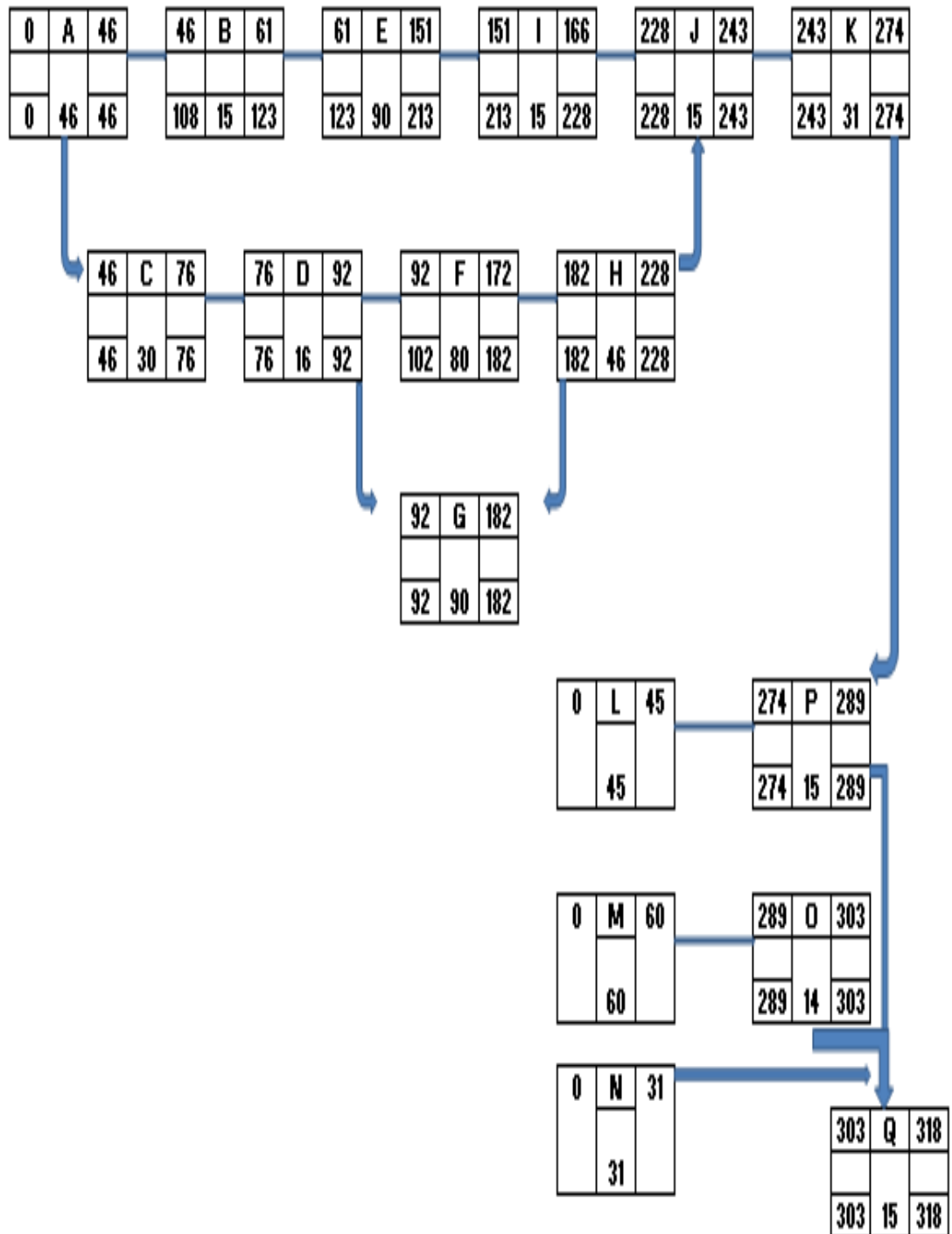


Figure 4. 4: CPM project main iteration

Crashing strategy:

Since terminating a non-critical activity does not affect the total project timeline, we crash. The activities that will be crashed on the critical path are ordered by increasing cost. Applying the cost slope equation below, crash cost per day (\$) is calculated and provided in Table 4.3.

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

Critical Path is; A→C→D→G→H→J→K→P→O→Q

Crushing an activity depends on: Firstly the activity has to be crushable and on the critical path shown in Table 4.3, if we fulfill these two conditions then we chose the activity with the cheapest cost, and to what level a reduction in project time and expense is required. To shorten the project's duration, the activity cost slope must be examined to determine which activities may be decreased for the least cost; this will be done in stages called iterations until the complete number of days to be crashed is reached Note that. The crashing is implemented until the duration has been crashed by more than 35% of the total duration of the project or approximately 203 Days, and the budget limit of the project is 12% of the total cost of the project which is equal approximately to 18,430\$. The iterations are completed in 10 iterations given in the following pages. Table 4.4 below displays the output of the manual solution method.

The project crashed from 318 days to 203 days, and the cost of the project was 153,590\$ also increased to a cost of 170,800 dollars. It minimizes the duration of the project by more than 35% and minimizes the cost by approximately 11,7%. This is fitting our target by more than 35% of the total duration of the project to 203 days, and the budget limit of the project is 12% of the total cost of the project which is equal to approximately 18,430 \$.

Normal cost = 153,590 \$

Crashing cost = 360+ 1000+ 1533.333+ 1000+ 1850+ 800+ 1000+ 3500+ 4666.667+ 1500= 17,210 \$

→ (153,590 \$+17,210\$) = 170,800 \$

1st iteration:

The first cashable activity option on the critical path is Q activity with the smallest crash cost of 45\$ per day decreasing the activity duration from (15 to 7 days), and the crash cost increased from (440\$ to 800\$).

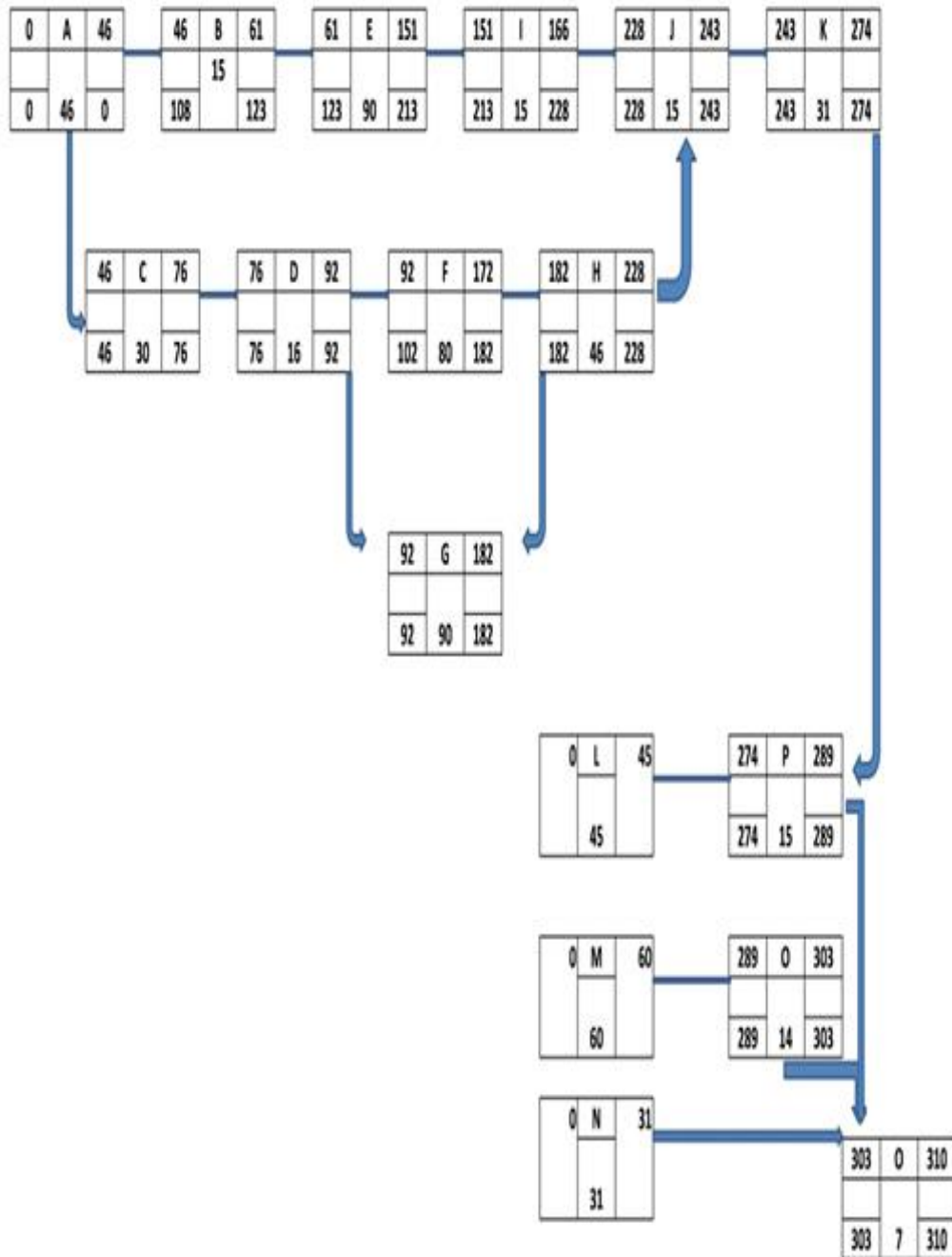


Figure 4. 5: The first crashing iteration

2nd iteration:

The second cashable activity option on the critical path is C activity with the 2nd smallest crash cost smallest crash time decreased from (30 to 15 days), and the crash cost increased from (3000\$ to 4000\$).

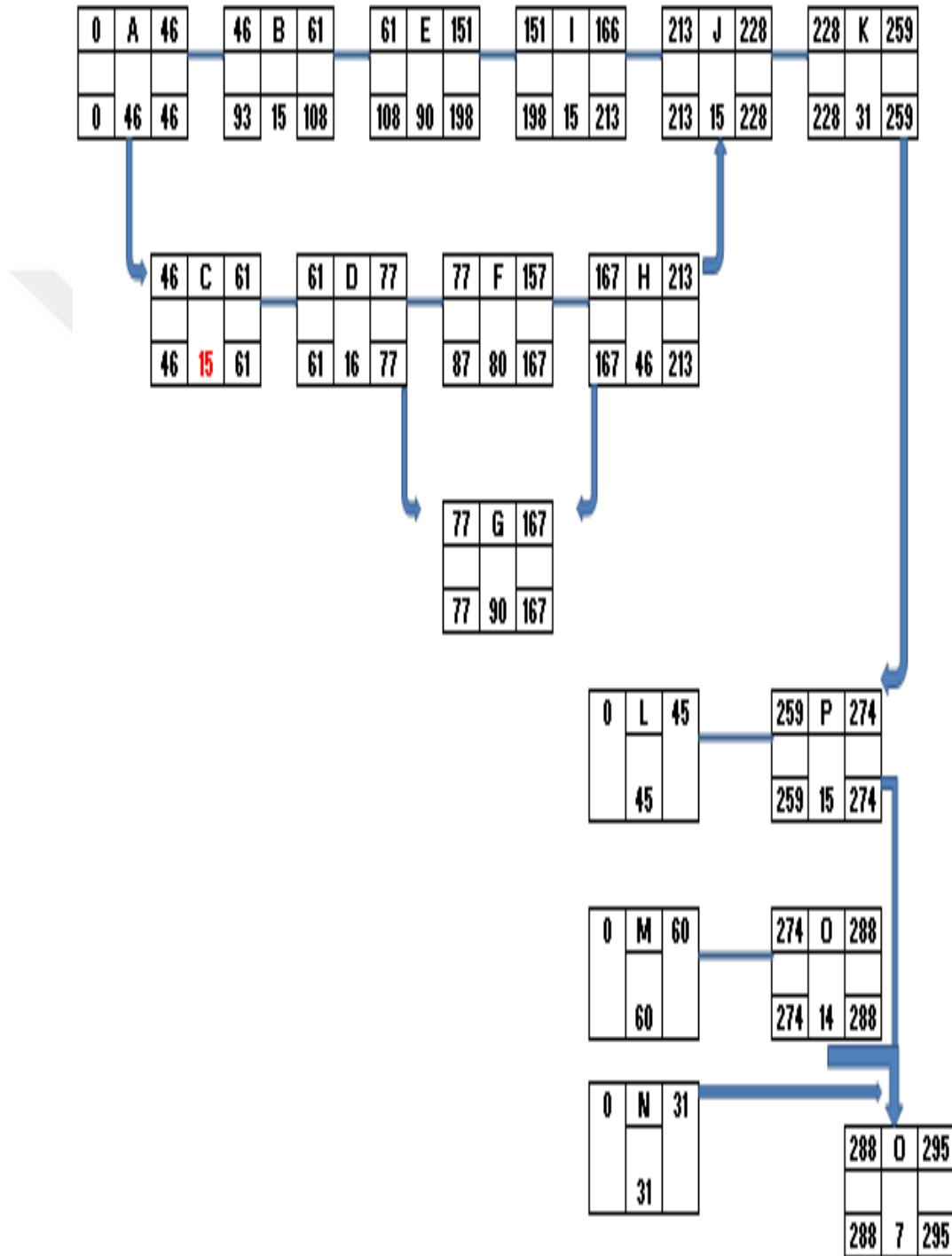


Figure 4. 6: The second crashing iteration.

3rd iteration:

The third crashable activity option on the critical path is K activity with the 3rd smallest crash cost smallest crash time decreased from (31 to 15 days), and the crash cost increased from (3200\$ to 5500\$).

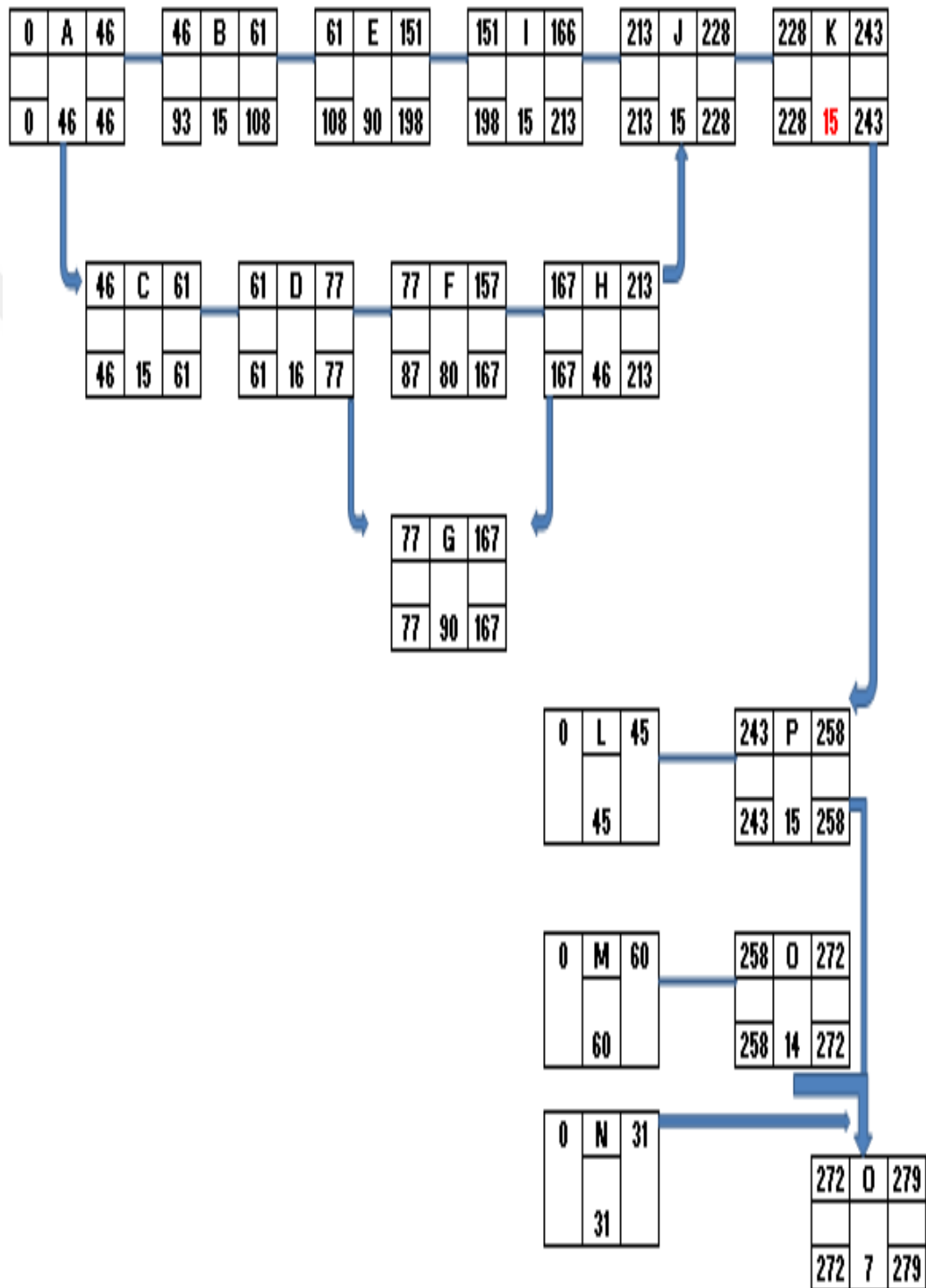


Figure 4. 7: The third crashing iteration.

4th iteration:

The fourth crashable activity option on the critical path is D activity with the 4th smallest crash cost smallest crash time decreased from (16 to 7 days), and the crash cost increased from (3500\$ to 4500\$).

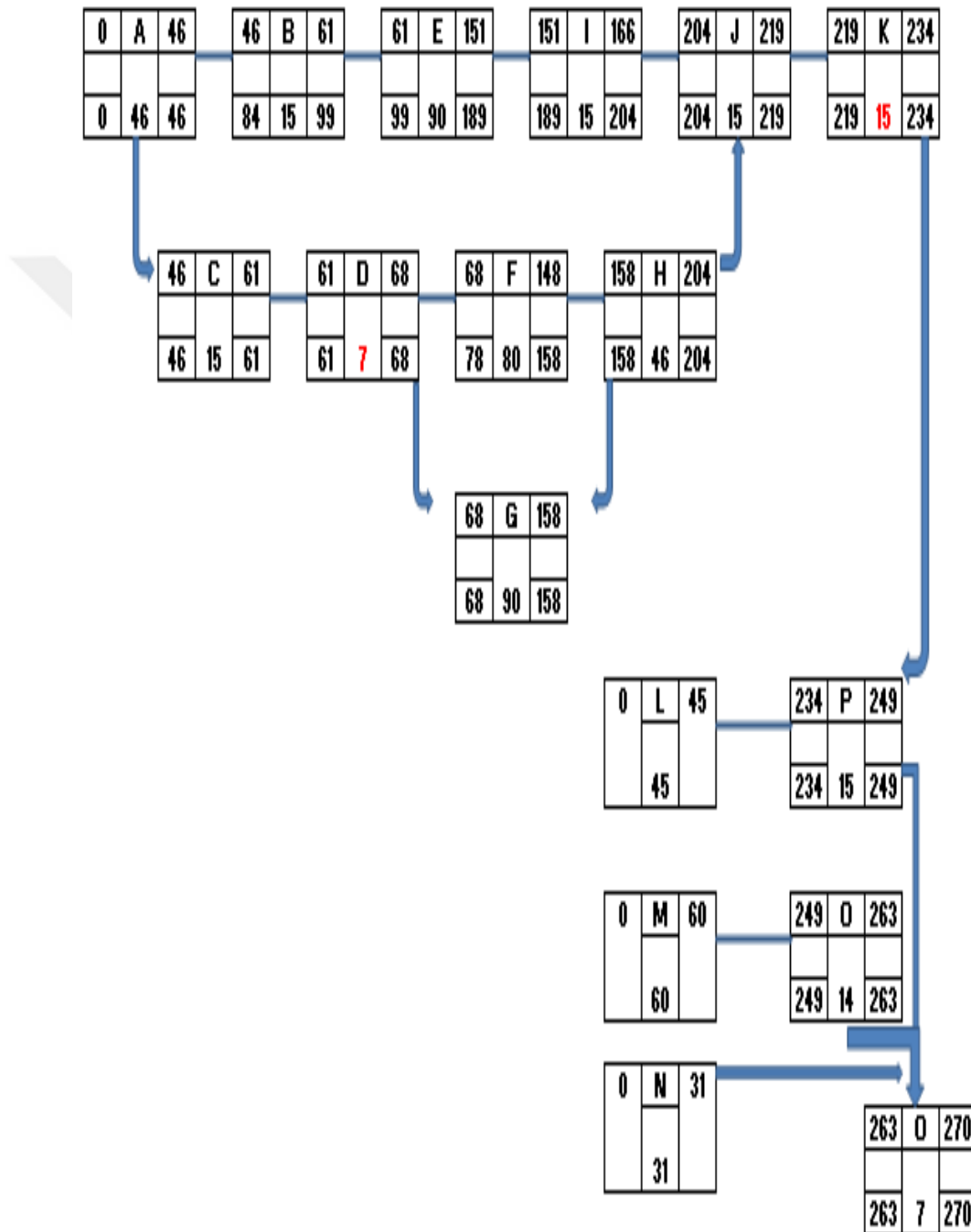


Figure 4. 8: The fourth crashing iteration.

5th iteration:

The fifth crashable activity option on the critical path is A activity with the 5th smallest crash cost smallest crash time decreased from (46 to 30 days), and the crash cost increased from (7650\$ to 9500\$).

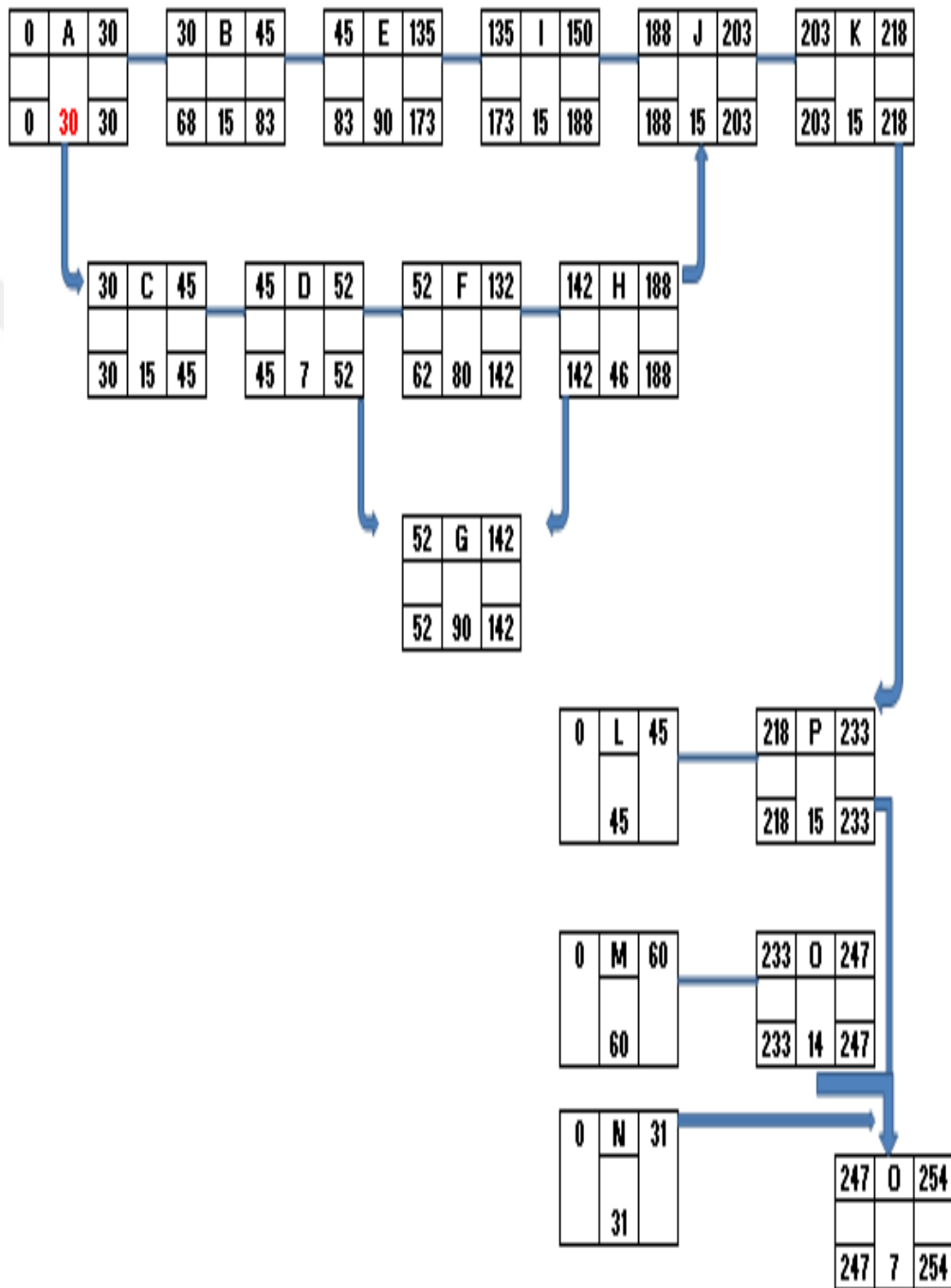


Figure 4. 9: The fifth crashing iteration.

6th iteration:

The sixth crashable activity option on the critical path is O activity with the 6th smallest crash cost smallest crash time decreased from (14 to 7 days), and the crash cost increased from (2200\$ to 3000\$).

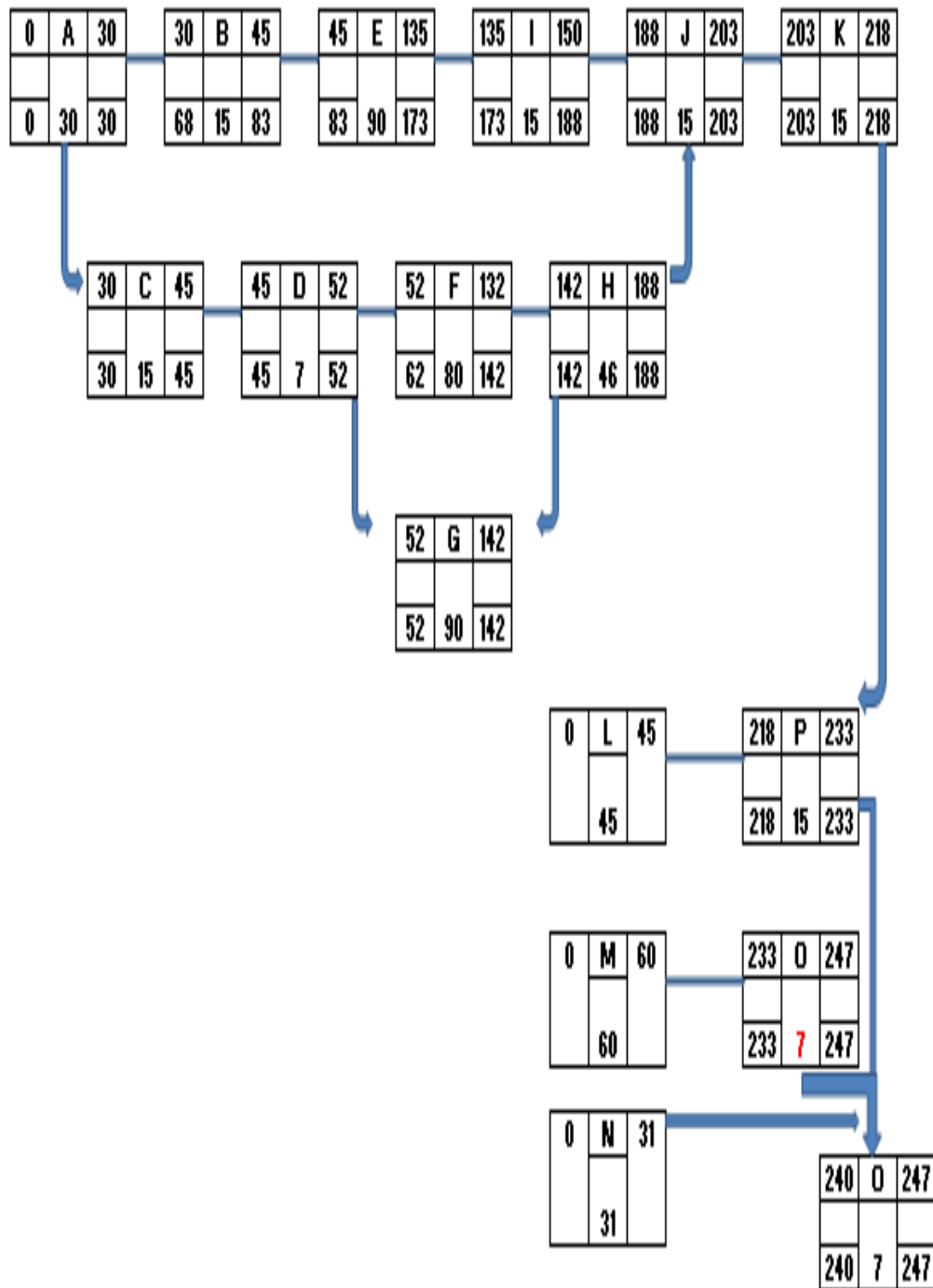


Figure 4. 10: The sixth crashing iteration.

7th iteration:

The seventh crashable activity option on the critical path is for P activity with the 7th smallest crash cost smallest crash time decrease from (15 to 7 days), and the crash cost increase from (4000\$ to 5000\$).

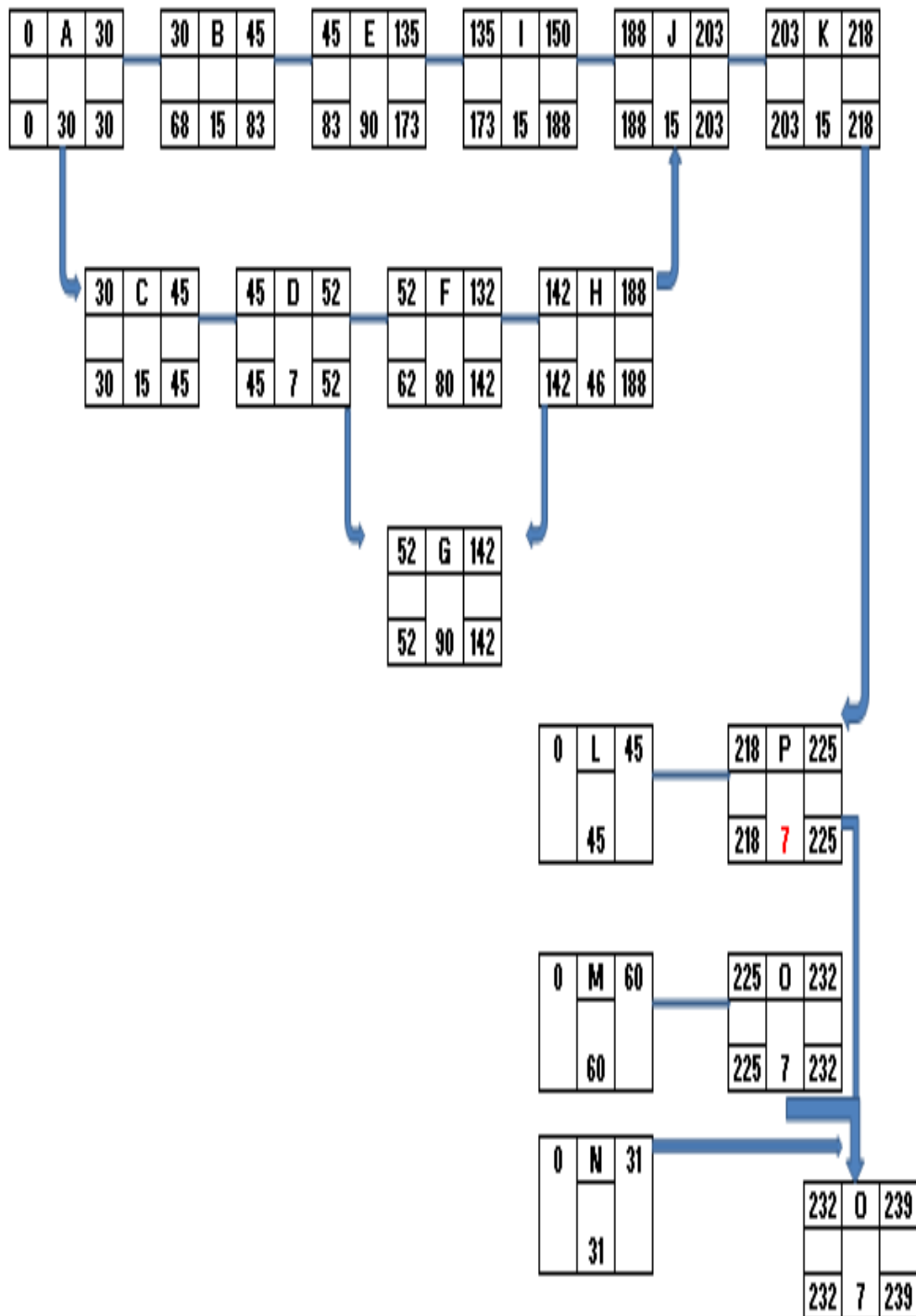


Figure 4. 11: The seventh crashing iteration.

8th iteration:

The eighth crashable activity option on the critical path is H activity with the 8th smallest crash cost smallest crash time decrease from (46 to 30 days), and the crash cost increase from (11000\$ to 15000\$).

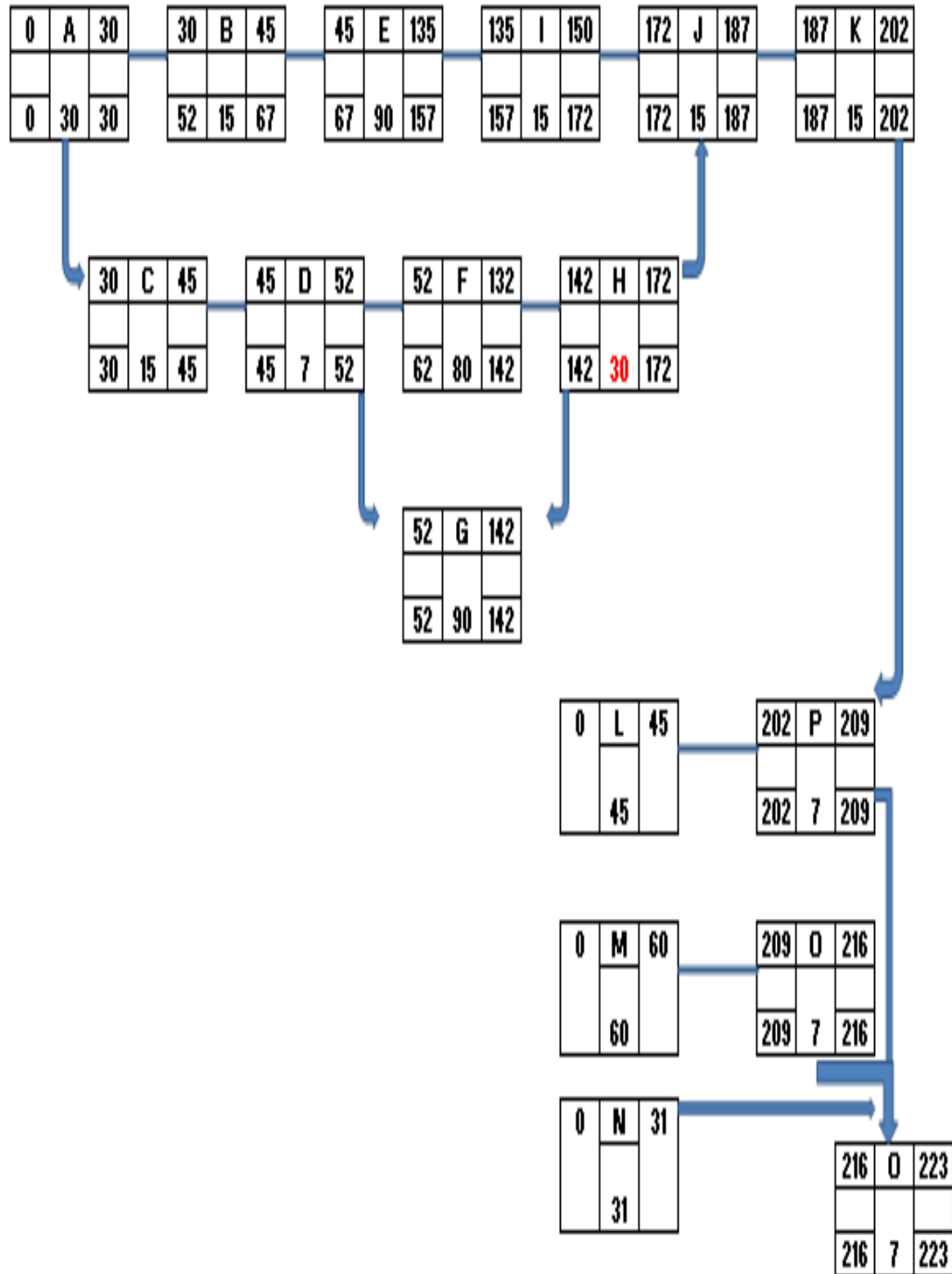


Figure 4. 12: The eighth crashing iteration.

9th iteration:

The ninth cashable activity option on the critical path is G activity with the 9th smallest crash cost smallest crash time decreased from (90 to 80 days), and the crash cost increased from (30000\$ to 32333\$).

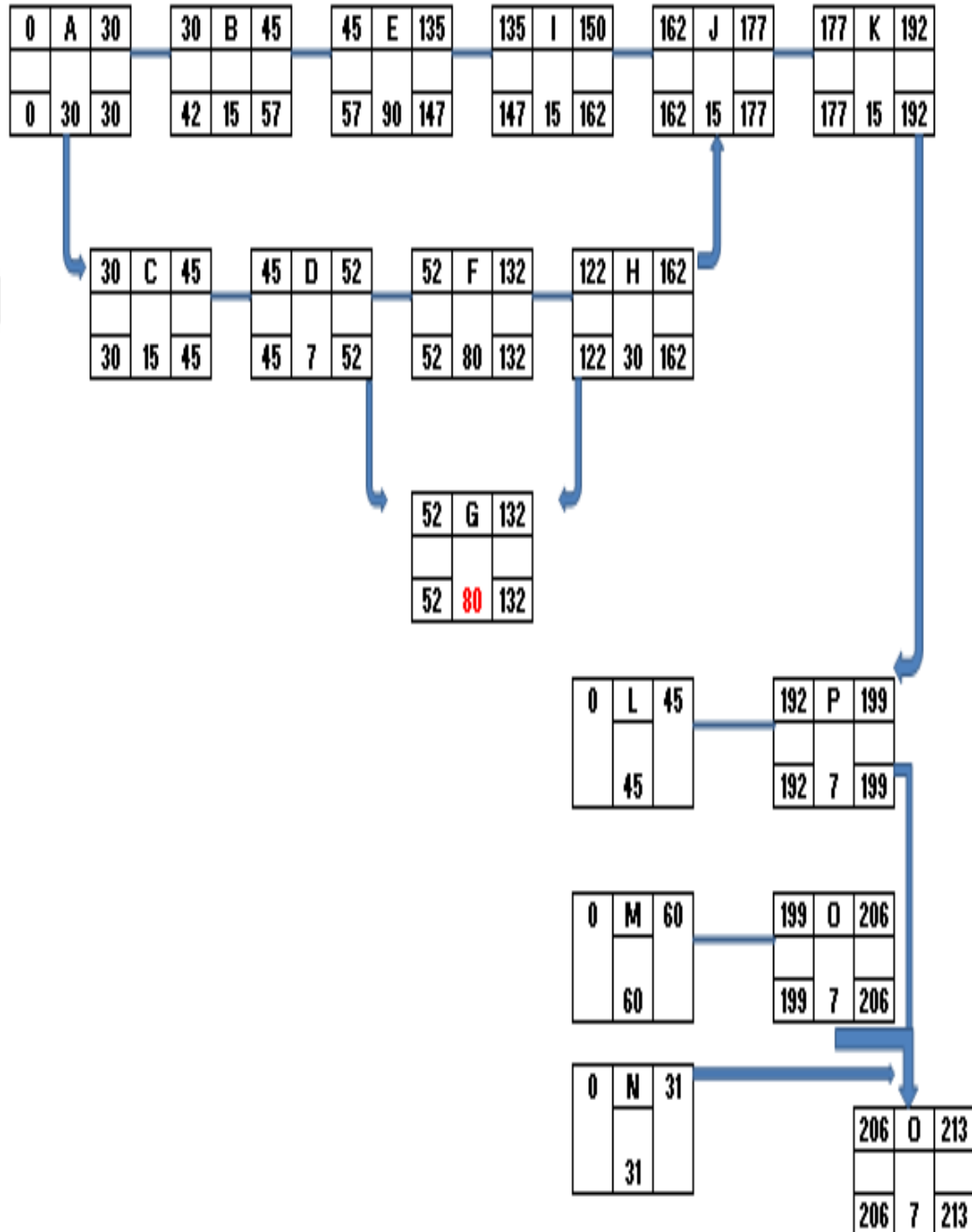


Figure 4. 13: The ninth crashing iteration.

10th iteration:

The last two cashable activity option on the critical path is for F, and G activity with the 10th smallest crash cost and smallest crash time decrease from (80 to 70 days), and the crash cost increase for the G option from (30000\$ to 32333\$), and F option from (20500\$ to 22000\$).

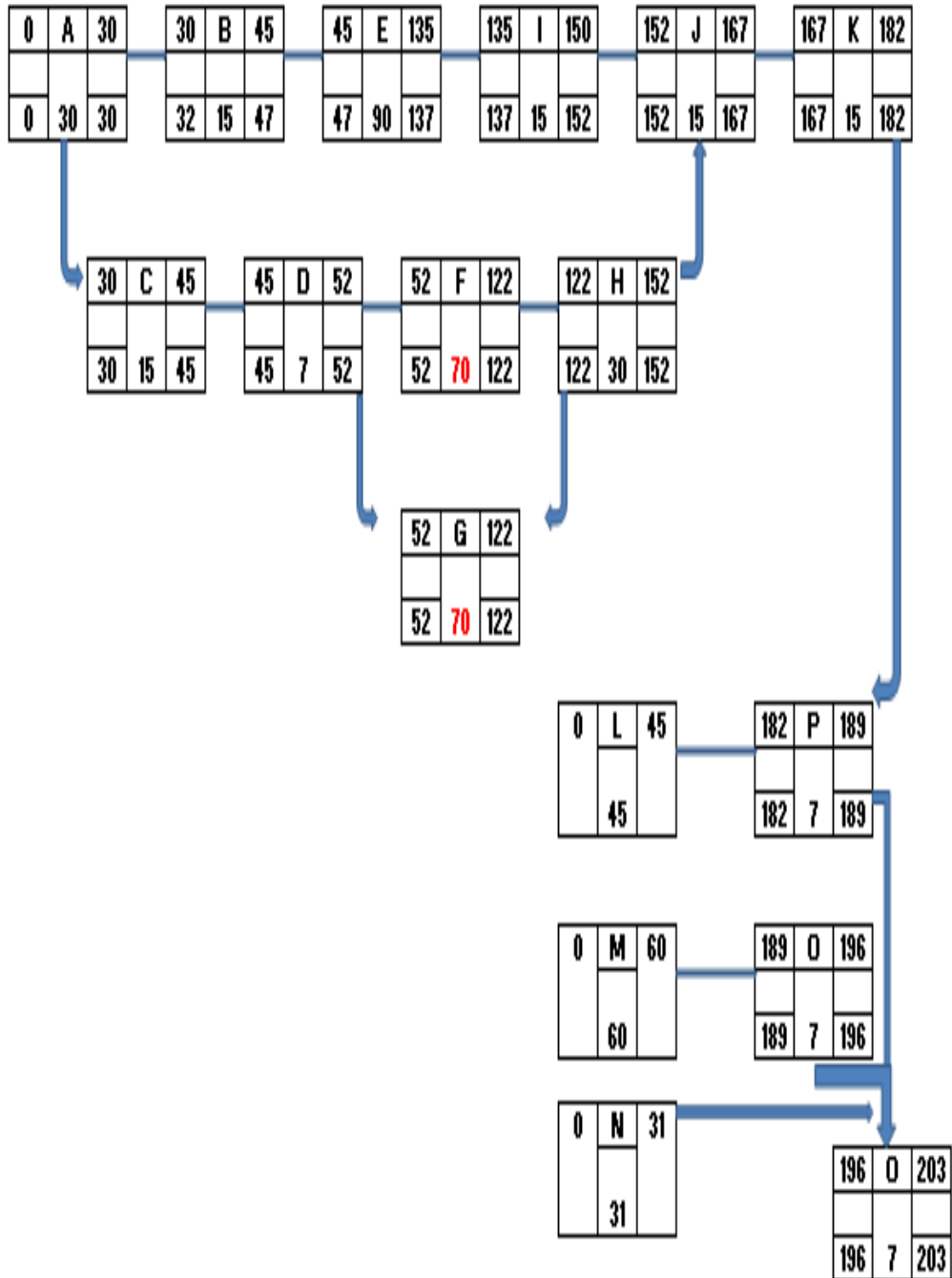


Figure 4. 14 The tenth crashing iteration.

Table 4.4: Manual solution crashing result

Activity	Activity Time Before Crashed (Days) t_{IN}	Activity Time After Crashed (Days) t_{IC}	Crashed duration $t_{IN} - t_{IC}$	Crashed Cost (\$/day)	Total Crash Cost for Each Activity (\$)
A	46	30	16	115.625	1850
B	15	15	0	150	0
C	30	15	15	66.6667	1000
D	16	7	9	111.1111	1000
E	90	90	0	166.6667	0
F	80	70	10	150	1500
G	90	70	20	233.3333	4666.6667
H	46	30	16	218.75	3500
I	15	15	0	687.5	0
J	15	15	0	400	0
K	31	15	16	95.83333	1533.3333
L	45	45	0	66.6667	0
M	60	60	0	73.3333	0
N	31	31	0	41.6667	0
O	14	7	7	114.286	800
P	15	7	8	125	1000
Q	15	7	8	45	360
SUM				Total cost:	17,210\$

Normal cost = 153,590 \$

Crashing cost = 360+ 1000+ 1533.333+ 1000+ 1850+ 800+ 1000+ 3500+ 4666.667

+ 1500= 17,210 \$

→ (153,590 \$+17,210\$) = 170,800 \$

4.4 Optimum Crashing Solution of Mathematical Model with Excel

Once we have the manual solution to the project, the mathematical model is coded for the Excel solver. The components for this approach are as follows;

Mathematical Model:

Indices:

i : activity index, $i = 1, 2, \dots, q$

$N+1$ is the activity indicating the finish of the project

Since we have seventeen activities in the project. A code was used to designate an activity, such as (X1 ..., X17.)

Parameters:

t_{iN} : Normal duration of activity i ,

t_{iC} : Crashed duration of activity i ,

C_{iN} : Normal cost of activity i ,

C_{iC} : Crashed cost of activity i ,

C_i : Cost slope (crash cost per period) for activity

TD : Target Duration of the Project

Decision Variables:

X_i : the early finish time for activity i ; $i: a, b, \dots, q$

Y_i : crashed duration of activity i (number of periods it is crashed); $i: A, B, \dots,$

Objective function;

$$\begin{aligned} \text{Min } Z = \sum C_i Y_i = & 116Y_A + 150Y_B + 67Y_C + 111Y_D + 167Y_E + \\ & 150Y_F + 233Y_G + 219Y_H + 688Y_I + 400Y_J + 96Y_K + 67Y_L + \\ & 73Y_M + 42Y_N + 114Y_O + 125Y_P + 45Y_Q \end{aligned}$$

Subject to.

Days that can activity is crashed (Maximum reduction constraints):

$Y_A \leq 16, Y_B \leq 8, Y_C \leq 15, Y_D \leq 9, Y_E \leq 30, Y_F \leq 40, Y_G \leq 30, Y_H \leq 16, Y_I \leq 8, Y_J \leq 5, Y_K \leq 24, Y_L \leq 15, Y_M \leq 30, Y_N \leq 24, Y_O \leq 7, Y_P \leq 8, Y_Q \leq 8$

Non-negativity constraints:

$X_i \geq 0, i: a, b, \dots, q \rightarrow X_a, X_b, \dots, X_q \geq 0,$

$Y_i \geq 0, i: A, B, \dots, Q \rightarrow Y_A, Y_B, \dots, Y_Q \geq 0,$

Start Time Constraints:

$X_a = 0,$

For 1st event, $X_a - X_{start} + Y_A \geq 46$

For 2nd event, $X_b - X_a + Y_B \geq 15$

For 3rd event, $X_c - X_a + Y_C \geq 30$

For 4th event, $X_d - X_c + Y_D \geq 16$

For 5th event, $X_e - X_b + Y_E \geq 90$

For 6th event, $X_f - X_d + Y_F \geq 80$

For 7th event, $X_g - X_d + Y_G \geq 90$

For 8th event, $X_h - X_f + Y_H \geq 46$

For 8th event, $X_h - X_g + Y_H \geq 90$

For 9th event, $X_i - X_e + Y_I \geq 15$

For 10th event, $X_j - X_i + Y_j \geq 15$

For 10th event, $X_j - X_h + Y_J \geq 46$

For 11th event, $X_k - X_j + Y_K \geq 31$

For 12th event, $X_o - X_p + Y_O \geq 14$

For 12th event, $X_o - X_m + Y_o \geq 60$

For 13th event, $X_p - X_k + Y_P \geq 15$

For 13th event, $X_p - X_l + Y_P \geq 45$

For 14th event, $XQ - X_o + YQ \geq 15$

For 14th event, $XQ - X_n + YQ \geq 31$

For 14th event, $XQ - X_p + YQ \geq 15$

For 15th event, $X_o + X_{finish} \geq 0$

Target Duration:

$X_q \leq 203$

Solution of the linear programming Using Excel Solver

The objective function and constraints are entered into the excel sheet as shown in Figure 4.16 to solve the LP model using Excel Solver. Then, it is necessary to configure Solver parameters and restrictions. It will go through the specifics of the methods in depth while referring to the spreadsheet fields

LP's goal function is represented by the target cell in the set target cell issue. The entire cost of the project is kept to a bare minimum. The LP decision variables are defined by changing the cells in the LP decision matrix. Solver attempts to determine the ideal values that fulfill all of the criteria while also minimizing the overall project cost by varying the number of days that each activity is crashed.

The codes of the Excel are provided in Figure 4.16. The first constraint assures that the first activity starts at a certain time ($t=0$)

$$AL25 = AN25$$

During the optimization process, the set of crashing days takes into account project reality, with negative crashing days not being permitted. It necessitates the existence of non-negative choice factors. Also, the activity durations are greater than or equal to the start time of the predecessor activities (Start Time Constraints)

$$AL26:AN49 \geq AL26:AN49$$

The max time for an activity to be crashed is limited (Maximum reduction constraints) and the project completion time will be less than or equal to a previously set completion date; (Li, Shao, & Zelbst, 2012)

$$AL7:AL24 \leq AN7:AN24$$

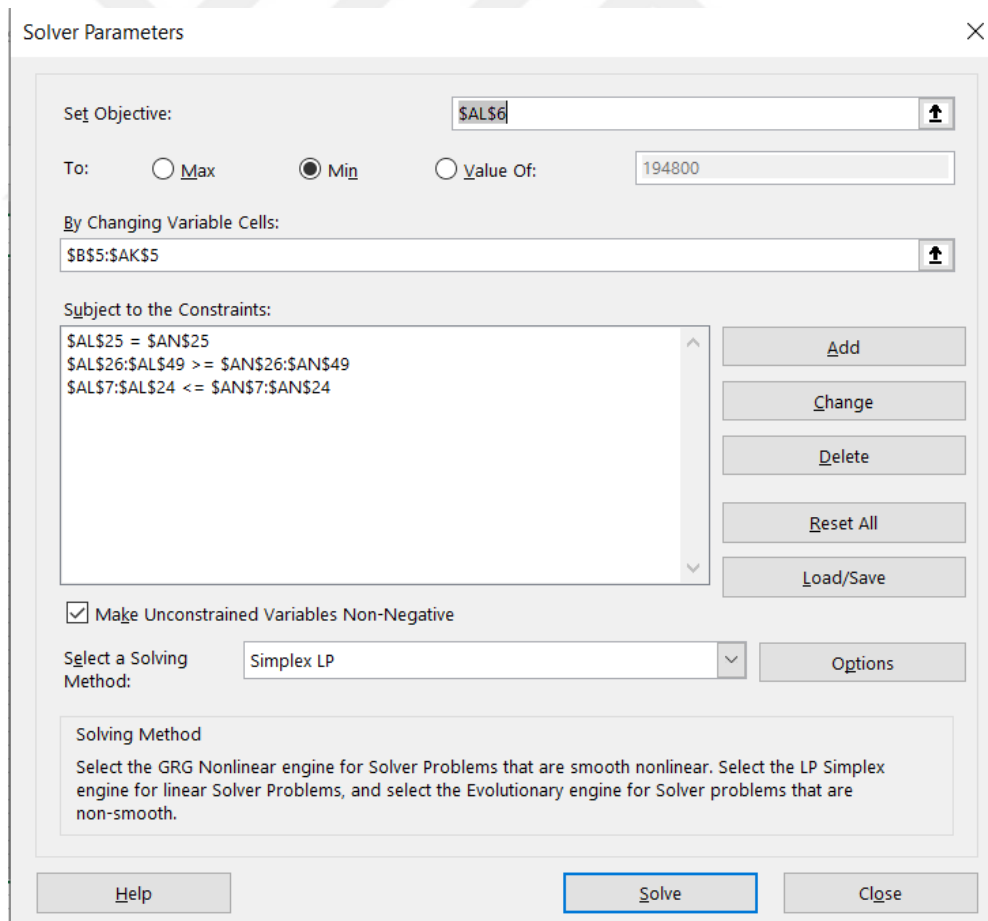


Figure 4. 16: Excel solver defining objective function and constraints

After the run of the solver, the output for X_i and Y_i is obtained together with the total cost of 14,910 dollars and crashing for 115 days to reach the Project Target Duration of 203 days.

The optimum solution provided by the solver crashed 10 activities; A, C, D, F, G, H, K, O, P, and Q by crashing them in 16,15,9,2,12,16,24,7,8,8 Days Respectively.

Table 4. 5: Solution using Excel Solver (Y_i and crashing costs).

Crashed durations (Days) (Y_i)		Crash cost for each activity (\$/day)	Total Crashed Cost for each Activity (\$)
YA	16	115.625	1850
YB	0	150	0
YC	15	66.6667	1000
YD	9	111.111	1000
YE	0	166.6667	0
YF	2	150	300
YG	12	233.3333	2800
YH	16	218.75	3500
YI	0	687.5	0
YJ	0	400	0
YK	24	95.8333	2300
YL	0	66.6667	0
YM	0	73.3333	0
YN	0	41.6667	0
YO	7	114.286	800
YP	8	125	1000
YQ	8	45	360
		Total cost :	14,910\$

Normal cost = 153,590 \$

Another output variable by the optimum solution is the earliest finish time of each activity shown in Table 4.6.

Table 4. 6: Solution using Excel Solver (X_i : Early finish durations).

Earliest finish time for each activity	
Xa	30
Xb	55
Xc	45
Xd	52
Xe	145
Xf	130
Xg	130
Xh	160
Xi	160
Xj	175
Xk	182
Xl	182
Xm	189
Xn	196
Xo	196
Xp	189
Xq	203

To sum up the optimization approach; the project was considered to be finished in 318 days without crashing with a cost of 153,590 dollars in the beginning. By applying the linear programming approach and solved by Excel solver the project is finished in 203 days target with a total crashing cost of 14,910 dollars. Using this method, the project's duration is reduced by more than 35 %, to less than 10 % increase in the total Project cost. The total cost of the project happened to be 168,500 dollars with this solution.

4.5 Comparative Analysis of Results

To estimate the Minimum Crashing Cost to shorten the project's duration to achieve our target. We have used two methods:

- A manual solution using CPM.
- A linear programming solution applying a model on an Excel solver.

The primary goal of our estimates is to shorten the project's length while keeping the crashing costs to a bare minimum. After Applying the two methods we got two different results for the same target duration of 203 days.

After applying the manual solution using the critical path method, we found the results shown in Table 4.4. As a result of the Manual solution using CPM, the project was considered to be finished in 318 days without crashing with the cost of the project is 153,590 dollars. By applying the critical path method for ten iterations and crashing the critical activities in the project, the project is shortened to be finished in 203 days which fits our crashing target with 17,210 dollars increase in the cost. Thus, the total project cost has increased from 153,590 dollars to 170,800 dollars. Using this method, the project's duration is reduced by more than 35 %, with more than 11 % increase in the total cost.

On the other hand, Excel solver provided a better cost for the same duration in a shorter time. With this solution, the crashing cost of the project happened to be 14,910 dollars for 203 days and the total project cost is resulted as 168,500 dollars. This corresponds to a 9.7% cost increase. The comparisons for the crashed activities and the costs are provided in Table (4.7)

Table 4. 7: Comparative results of Excel and manual approach

	Crashed durations (Days) (Yi)		Crash Cost for each Activity (\$)	
	Excel	Manual	Excel	Manual
YA	16	16	1850	1850
YB	0	0	0	0
YC	15	15	1000	1000
YD	9	9	1000	1000
YE	0	0	0	0
YF	2	10	300	1500
YG	12	20	2800	4666.6667
YH	16	16	3500	3500
YI	0	0	0	0
YJ	0	0	0	0
YK	24	16	2300	1533.3333
YL	0	0	0	0
YM	0	0	0	0
YN	0	0	0	0
YO	7	7	800	800
YP	8	8	1000	1000
YQ	8	8	360	360
Total crashing cost :			14,910\$	17,210\$

The resulting crashed days in Figure 4.18 show variation values of days in some activities and equal values of days in some other activities. The resulting crashed cost in Figure 4.19 below shows also different values of cost in some activities and equal values of cost in some other activities. Crashing by CPM shows paying an extra cost in some activities like (F_G_H) and paying the lower cost in (K) activity would reaching an increase the normal cost of 17,210\$ to reach the total cost of

170,800\$ while Crashing by LP shows paying the lower cost in some activities like (F_G_H) and higher reaching an increase the normal cost with 14,910\$ to reaching the total cost 168,500 \$ which is less than the CPM's result crashed cost.

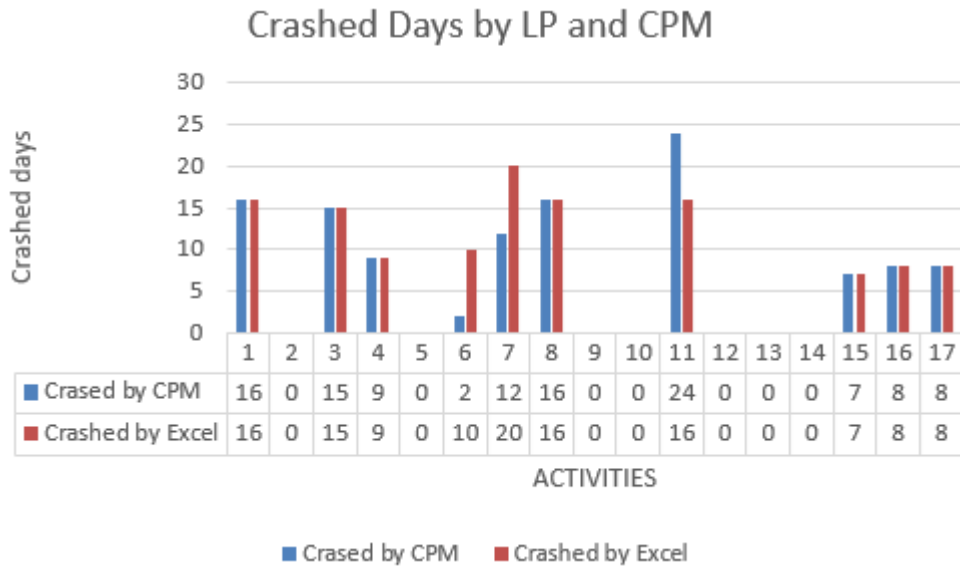


Figure 4. 18: Comparative results (Crashed days of LP and CPM)

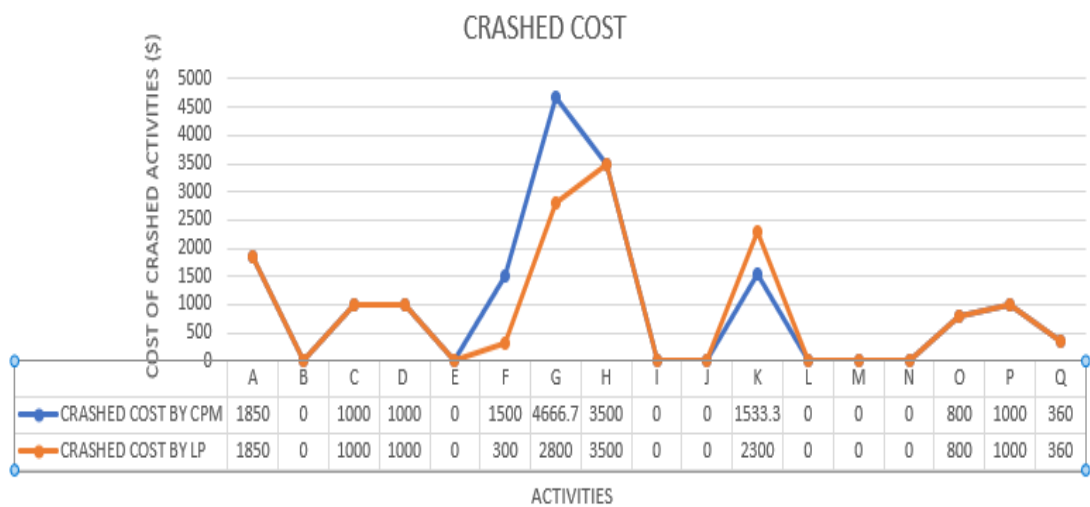


Figure 4. 19: Comparative results (Crashed cost of LP and CPM)

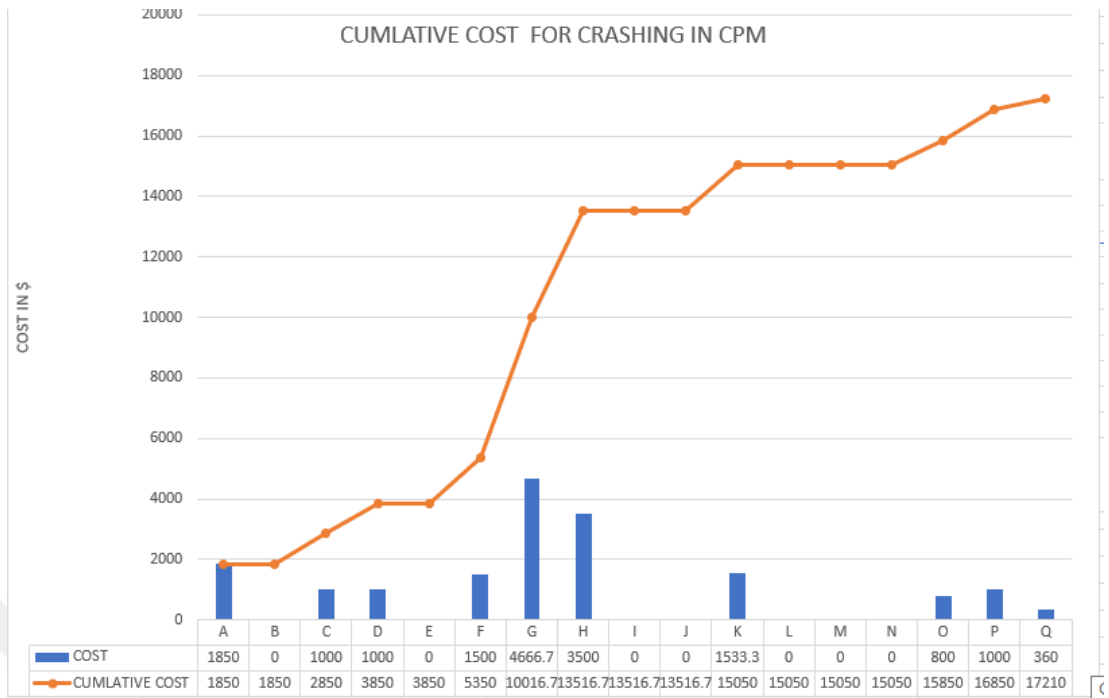


Figure 4. 20: Cumulative costs for crashing in CPM

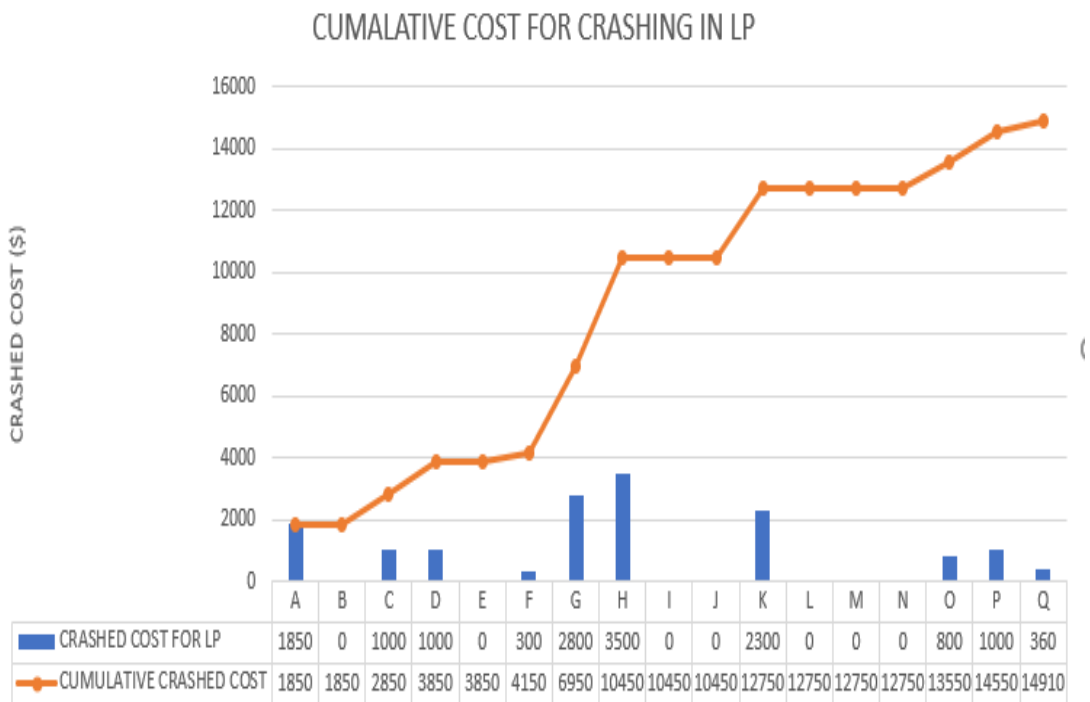


Figure 4. 21: Cumulative costs for crashing in LP

5 CONCLUSION

When a project falls behind schedule, the costs associated with it increase. Furthermore, severe changes in the currency exchange rates affect negatively the construction business sector. This rapid change in currency exchange rates affects the costs of construction materials. Hence, these fluctuations in the foreign currency rate have a vital effect on the success or failure of the profit targets of construction institutions. As the duration of the project increases, it is more subject to these fluctuations. On the other hand, shortening the duration also requires a certain amount of budget increase. Hence, the objective of this study is to find the best trade-off for this problem

In this study, a construction project of a cooling tower in Damascus, Syria was analyzed using a project management model. Initially, normal, crashing costs and durations of the project activities were obtained. Two different approaches are used to crash the project duration for 35% (203 days).

In the CPM method with a manual approach, Normal completion time (318 days) and critical activities (A, C, D, G, H, J, K, P, O, and Q) were determined. Afterward, manual crashing in CPM for every smallest cost crushable activity on the critical pathway until we fulfill our duration and cost target, first crashing iteration was crashing activity Q we decreased from (15 to 7 days), and the crash cost increased from (440\$ to 800\$). Secondly C activity with the 2nd smallest crash cost, its duration was decreased from (30 to 15 days), and the crash cost increased from (3000\$ to 4000\$). The third iteration comes K activity with the 3rd smallest crash cost, its duration was decreased (31 to 15 days), and the crash cost increased from (3200\$ to 5500\$). Fourth iteration D activity with the 4th smallest crash cost, its duration was decreased from (16 to 7 days), and the crash cost increased from (3500\$ to 5033\$). Fifth iteration A activity with the 5th smallest crash cost, its duration was decreased from (46 to 30 days), and the crash cost increased from (7650\$ to 9500\$). The sixth iteration comes O activity with the 6th smallest crash cost, its duration decreased from (14 to 7 days), and the crash cost increased from (2200\$ to 3000\$). The seventh iteration was for P activity with the 7th smallest crash cost, its duration decreased from (15 to 7 days), and the crash cost increased

from (4000\$ to 5000\$). The eighth iteration was H activity with the 8th smallest crash cost, its duration was decreased from (46 to 30 days), and the crash cost increased from (11000\$ to 15000\$). The ninth iteration was G activity with the 9th smallest crash cost, its duration was decreased from (90 to 80 days), and the crash cost increased from (30000\$ to 32333\$). The last iteration was with F, G activity with the 10th smallest crash cost, their duration was decreased from (80 to 70 days), and the crash cost increase for G option from (30000\$ to 32333\$), and F activity from (20500\$ to 22000\$).

The result of the manual crashing is a significant reduction of nearly 115 days in the projected interval crashing the project duration from 318 days to 203 days which is approximately a 35% reduction of the total duration and an increase in the project cost by Nearly 11,2% of the total cost of the project

For the second approach, a mathematical model to be solved by an Excel solver was developed and used for crashing projects for critical activities resulting in the same reduction of the duration. This approach provided a better cost approximately 9.7% of the total cost of the project saving 2300\$ with this solution.

The manual solution took one hour to reach the 203 days whereas the Excel is coded in one hour and the results are obtained in minutes. Although the preparation time for the Excel solution is longer, it is open to discussing various alternative scenarios of different due dates. These results displayed that using an Excel solver supports finding the optimum crashing cost in a shorter period.

This study is implemented to assist project managers in the present or the future with more accurate scheduling solutions by crashing some activities in their projects. This hands-on effort will assist in reducing project duration at the lowest possible cost to fulfill the best profit in the construction business sector in the middle-developed or underdeveloped countries where their construction material affects the exchange rates as the operations and materiel depend on a foreign exchange-based activities.

Future studies can be applied for much more complex construction projects based on parallel activity or operations by providing mutual dependencies and their impact on the quality (Mahmoudi & Feylizadeh, 2017) and the risks of the project after being crashed for specific crash costs.

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