

Optical Soliton Solutions of a $(3 + 1)$ -Dimensional Nonlinear Schrödinger Equation with Cubic-Quintic-Septic Nonlinearity

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Abstract

In this paper, we studied the extended $(3+1)$ -dimensional NLSE that has cubic-quintic-septic nonlinearity term. To obtain meaningful results, we first defined the wave transform and applied it to the NLSE. We then derived the ordinary differential equation (ODE). We gave brief information about the new Kudryashov method (nKM) and applied it to our problem. Finally, we solved the system using Maple. Using these solutions, we obtained a bright soliton and then illustrated the graphics and analyzed the effect of the parameters of the equation on the graph of such a soliton solution.

Keywords: Optical soliton, Parameter effect, The new Kudryashov method, $(3+1)$ -dimensional NLSE

AMS Subject Classification (2020): 35C07; 35C08; 35G20; 35Q55; 35Q60

1. Introduction

The $(3+1)$ -dimensional nonlinear Schrödinger equation (NLSE) is a foundational model in the study of wave propagation and nonlinear dynamics in higher-dimensional systems. This equation extends the classical one-dimensional nonlinear Schrödinger equation (NLSE) to three spatial dimensions (x, y, z) and one temporal dimension (t) , thus rendering it a versatile tool for investigating complex physical phenomena in fields such as nonlinear optics, Bose-Einstein condensates, plasma physics and fluid dynamics. A plethora of real-world phenomena, including ultrashort laser pulses in bulk media in three-dimensional traps, necessitate modelling that extends beyond the confines of the classical $(1+1)$ -dimensional nonlinear Schrödinger equation (NLSE) [1–6].

The $(3 + 1)$ -dimensional extension of the NLSE allows for the modeling of more complex spatiotemporal wave dynamics involving three spatial dimensions and one time dimension. Such formulations are crucial for understanding femtosecond pulse propagation, self-focusing, filamentation in Kerr media [7], and nonlinear

Received : 06-04-2025, Accepted : 01-07-2025, Available online : 30-07-2025

(Cite as "M. F. Uçar, Optical Soliton Solutions of a $(3 + 1)$ -Dimensional Nonlinear Schrödinger Equation with Cubic-Quintic-Septic Nonlinearity, 13(3) (2025), 116-125")



interactions in Bose–Einstein condensates [8]. Applications in quantum fluids and dusty plasmas further necessitate these multidimensional frameworks [9–19].

To more accurately model physical systems at high intensities, researchers have introduced higher-order nonlinear terms to the NLSE. The classical cubic nonlinearity accounts for Kerr effects, while the quintic term reflects saturation phenomena at high power levels. More recently, the septic nonlinearity has emerged as a key term in modeling nonlinear response in ultra-intense optical fields and advanced plasma systems [20, 21]. Incorporating such higher-order terms allows for a more comprehensive description of nonlinear dynamics, especially in regimes where lower-order approximations fail.

In parallel with the development of generalized models, a wide range of analytical and semi-analytical techniques have been proposed to extract exact or approximate solutions to (3 + 1)-dimensional NLSEs. The Kudryashov method and its refined versions, such as the new Kudryashov method (nKM), have been extensively used to handle equations with strong nonlinearities including quintic and septic terms [22–25]. Lie symmetry analysis has been successfully applied to fractional and time-dependent extensions of the NLSE, offering symmetry reductions and conservation laws [26]. In addition, the modified extended mapping method has yielded hyperbolic and rational function solutions for higher-dimensional wave equations [27].

Other approaches such as the first integral method [28], Hirota bilinear form, and homogeneous balance techniques have been implemented to generate bright, singular, breather, and rogue wave solutions in various physical settings. For example, vortex solitons in quantum fluids [29], chirped wave structures [30], and chaotic wave dynamics [31] have been reported in generalized (3 + 1)-dimensional NLSE-type models. Researchers have also investigated dual-wave and modulation instability phenomena in Kadomtsev–Petviashvili and Sasa–Satsuma type systems [32–37]. Studies involving fractional-order derivatives have further extended the applicability of NLSEs to nonlocal and memory-dependent media [38, 39].

Despite these advancements, most existing studies focus on cubic or cubic–quintic nonlinearities. Only limited attention has been given to models that simultaneously incorporate cubic, quintic, and septic nonlinear terms, especially in the presence of anisotropic dispersion and mixed derivative couplings. In paper [20] Wang presented a novel (3 + 1)-dimensional Schrödinger equation

$$iB_t - (B_{xx} + B_{yy} + B_{zz} + 2B_{xy} - 2B_{xz} - 2B_{yz}) + 2|B|^2B = 0,$$

that has cubic nonlinear term and in paper [21] Wang et.al. presented more general form than the previous equation that has cubic-quintic nonlinear term

$$iA_t - (b_1A_{xx} + b_2A_{yy} + b_3A_{zz} + 2b_4A_{xy} - 2b_5A_{xz} - 2b_6A_{yz}) + a_1|A|^2A + a_2|A|^4A = 0.$$

Despite these extensive developments, the combined effects of septic nonlinearity and anisotropic dispersion in (3+1)-dimensional systems remain unexplored, a gap our work addresses through. In this paper, we focus on more general form than the equation in [21], which has cubic-quintic-septic law nonlinearities as

$$iC_t - (\alpha_1C_{xx} + \alpha_2C_{yy} + \alpha_3C_{zz} + 2\alpha_4C_{xy} - 2\alpha_5C_{xz} - 2\alpha_6C_{yz}) + \beta_1|C|^2C + \beta_2|C|^4C + \beta_3|C|^6C = 0. \quad (1.1)$$

where $C(x, y, z, t)$ denotes the complex wave profile, the constant coefficients $\alpha_i (i = 1, 2, 3)$ are group velocity dispersions in $x, y,$ and z respectively, $\alpha_i (i = 4, 5, 6)$ are couple spatial dimensions, the factor of 2 arise from symmetry in mixed derivatives, β_1 is Kerr nonlinearity, β_2 is saturation of Kerr effect at high intensities and β_3 is higher-order nonlinearity. If the coefficients $\alpha_4 = \alpha_5 = \alpha_6 = \beta_2 = \beta_3 = 0$ in Eq.(1.1), then it is clear that the such equation is identical to (3+1)-dimensional Schrödinger equation. If $\beta_3 = 0$, then the equation is exactly the considered equation in [21]. While previous studies [20, 21] focused on cubic and cubic-quintic nonlinearities, the inclusion of septic nonlinearity in this work allows for a more comprehensive modeling of higher-order nonlinear effects, which are critical in certain optical regimes. Cubic-quintic terms are common in optics, but septic nonlinearity becomes significant in regimes with intense fields (e.g., high-power lasers), where lower-order approximations fail. Our aim is to explore how septic nonlinearity affects soliton stability, shape, and propagation in higher dimensions using nKM.

The structure of the paper is organized as follows: In Section 2.1, the mathematical analysis of Eq. (1.3) is detailed, focusing on the transformation of the nonlinear partial differential equation (PDE) into a nonlinear ordinary differential equation (NODE). Section 2.2 provides a concise overview of the methodology employed in the study. Section 3 is dedicated to presenting and discussing the results obtained. Finally, Section 4 concludes the paper by summarizing the key findings and their implications.

2. Material and methods

2.1 Deriving the NODE

The wave transform is a standard technique to reduce multidimensional nonlinear PDEs to ODEs in a single variable. So we first employ a complex wave transform:

$$C(x, y, z, t) = U(\xi)e^{i\Omega(x,y,z,t)}, \quad \Omega(x, y, z, t) = k_1x + k_2y + k_3z + \varpi t + \phi_0, \quad \xi = x + y + z - vt. \quad (2.1)$$

Here all parameter values are real and in detail, $U(\xi)$ is the real-valued function that represents the soliton's amplitude, ξ represents the new wave variable, k_1, k_2, k_3 the wave numbers, v the velocity and ω the frequency. Furthermore, $\Omega(x, y, z, t)$ is the phase component, and ϕ_0 is the phase constant. When we apply Eq. (2.1) into Eq. (1.1), the following imaginary and real parts of the equations are obtained respectively:

$$((4\alpha_4 - 4\alpha_5 + 2\alpha_1)k_1 + (4\alpha_4 - 4\alpha_6 + 2\alpha_2)k_2 + (-4\alpha_5 - 4\alpha_6 + 2\alpha_3)k_3 + v) \left(\frac{dU}{d\xi} \right) = 0, \quad (2.2)$$

$$\begin{aligned} &(-\alpha_1 - \alpha_2 - \alpha_3 - 2\alpha_4 + 2\alpha_5 + 2\alpha_6) \left(\frac{d^2U}{d\xi^2} \right) + U \left(\beta_3U^6 + \beta_2U^4 + \beta_1U^2 + \alpha_1k_1^2 \right. \\ &\left. + (2\alpha_4k_2 - 2\alpha_5k_3)k_1 + \alpha_2k_2^2 + \alpha_3k_3^2 - 2\alpha_6k_2k_3 - \varpi \right) = 0. \end{aligned} \quad (2.3)$$

From Eq. (2.2), the velocity v is obtained from the condition that the coefficient of $dU/d\xi$ in the imaginary part must vanish to ensure the consistency of the solution, so we obtain the formula for the velocity as

$$v = -2\alpha_1k_1 - 2\alpha_2k_2 - 2\alpha_3k_3 - 4\alpha_4k_1 - 4\alpha_4k_2 + 4\alpha_5k_1 + 4\alpha_5k_3 + 4\alpha_6k_2 + 4\alpha_6k_3. \quad (2.4)$$

Then, homogeneous balance principle [25] is employed to Eq. (2.3) to determine the balance constant. By considering the highest-degree nonlinear term U^7 and highest-order linear term U'' , the balance condition is given by $7M = M + 2$. Solving for M yields $M = \frac{1}{3}$. The reason for focusing on these terms is that they dominate the equation's behavior in certain asymptotic regimes or during wave localization, such as in soliton formation. Also balancing these two terms helps us determine the dominant interaction between dispersion (U'') and nonlinearity (U^7). However, since the applied method requires M to be a positive integer, an algebraic transformation is necessary to address this issue as

$$U(\xi) = H(\xi)^{1/3}. \quad (2.5)$$

Substituting Eq. (2.5) into Eq. (2.3) we derive

$$\begin{aligned} &9H^{\frac{10}{3}}\beta_2 + 9H^{\frac{8}{3}}\beta_1 - 3H(\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 - 2\alpha_5 - 2\alpha_6) \left(\frac{d^2H}{d\xi^2} \right) \\ &+ (2\alpha_1 + 2\alpha_2 + 2\alpha_3 + 4\alpha_4 - 4\alpha_5 - 4\alpha_6) \left(\frac{dH}{d\xi} \right)^2 \\ &+ 9H^2(\beta_3H^2 + \alpha_1k_1^2 + \alpha_2k_2^2 + \alpha_3k_3^2 + 2\alpha_4k_1k_2 - 2\alpha_5k_1k_3 - 2\alpha_6k_2k_3 - \varpi) = 0. \end{aligned} \quad (2.6)$$

So, we get two more constraints,

$$\beta_1 = \beta_2 = 0. \quad (2.7)$$

Substituting these constraints in Eq. (2.6), it can be written in modified form as

$$\begin{aligned} &-3H(\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 - 2\alpha_5 - 2\alpha_6) \left(\frac{d^2H}{d\xi^2} \right) + (2\alpha_1 + 2\alpha_2 + 2\alpha_3 + 4\alpha_4 - 4\alpha_5 - 4\alpha_6) \left(\frac{dH}{d\xi} \right)^2 \\ &+ 9H^2(\beta_3H^2 + \alpha_1k_1^2 + \alpha_2k_2^2 + \alpha_3k_3^2 + 2\alpha_4k_1k_2 - 2\alpha_5k_1k_3 - 2\alpha_6k_2k_3 - \varpi) = 0. \end{aligned} \quad (2.8)$$

Now, we apply the balance principle one more time to Eq. (2.8) by considering the terms H^4 and HH'' , and we find $M = 1$.

2.2 The nKM and its implementation

This part of the study presents a brief information about nKM method [40, 41]. Within the framework of nKM, the solution to Eq. (2.8) is formulated as:

$$H(\xi) = \sum_{i=0}^{M=1} A_i \Psi^i(\xi) = A_0 + A_1 \Psi(\xi), \quad A_1 \neq 0. \quad (2.9)$$

where the function $\Psi(\xi)$ is determined by the following differential equation:

$$\frac{d\Psi(\xi)}{d\xi} = \sqrt{\delta^2 \Psi^2(\xi) (1 - \eta \Psi^2(\xi))}, \quad (2.10)$$

with δ and η being arbitrary nonzero real parameters. The general solution to Eq.(2.10) is given by:

$$\Psi(\xi) = \frac{4L}{4L^2 e^{\delta\xi} + \eta e^{-\delta\xi}}, \quad (2.11)$$

where L is a free parameter that remains nonzero. Additionally, Eq. (2.11) can be rewritten in a hyperbolic form as:

$$\Psi(\xi) = \frac{4L}{(4L^2 + \eta) \cosh(\delta\xi) + (4L^2 - \eta) \sinh(\delta\xi)}. \quad (2.12)$$

Depending on the choice of η , Eq. (2.11) and Eq. (2.12) lead to different solitonic structures. When $\eta = 4L^2$, a bright soliton solution emerges:

$$\Psi(\xi) = \frac{1}{2L} \operatorname{sech}(\delta\xi),$$

whereas for $\eta = -4L^2$, the resulting solution corresponds to a singular soliton:

$$\Psi(\xi) = \frac{1}{2L} \operatorname{csch}(\delta\xi).$$

By substituting Eq. (2.9) into Eq. (2.8), utilizing Eq. (2.10), collecting powers of $\Psi(\xi)$ to form a polynomial and equating the coefficients of each power to zero results in a system of five algebraic equations associated with the coefficients of $\Psi^j(\xi)$, for $0 \leq j \leq 4$:

$$\left\{ \begin{array}{l} \Psi^0(\xi) : 9A_0^4\beta_3 + 9A_0^2\alpha_1k_1^2 + 9A_0^2\alpha_2k_2^2 + 9A_0^2\alpha_3k_3^2 + 18A_0^2\alpha_4k_1k_2 - 18A_0^2\alpha_5k_1k_3 - 18A_0^2\alpha_6k_2k_3 \\ \quad - 9\varpi A_0^2 = 0, \\ \Psi^1(\xi) : -3\delta^2 A_0 A_1 \alpha_1 - 3\delta^2 A_0 A_1 \alpha_2 - 3\delta^2 A_0 A_1 \alpha_3 - 6\delta^2 A_0 A_1 \alpha_4 + 6\delta^2 A_0 A_1 \alpha_5 + 6\delta^2 A_0 A_1 \alpha_6 \\ \quad + 36A_0^3 A_1 \beta_3 + 18A_0 A_1 \alpha_1 k_1^2 + 18A_0 A_1 \alpha_2 k_2^2 + 18A_0 A_1 \alpha_3 k_3^2 + 36A_0 A_1 \alpha_4 k_1 k_2 \\ \quad - 36A_0 A_1 \alpha_5 k_1 k_3 - 36A_0 A_1 \alpha_6 k_2 k_3 - 18\varpi A_0 A_1 = 0, \\ \Psi^2(\xi) : -\delta^2 A_1^2 \alpha_1 - \delta^2 A_1^2 \alpha_2 - \delta^2 A_1^2 \alpha_3 - 2\delta^2 A_1^2 \alpha_4 + 2\delta^2 A_1^2 \alpha_5 + 2\delta^2 A_1^2 \alpha_6 + 54A_0^2 A_1^2 \beta_3 + 9A_1^2 \alpha_1 k_1^2 \\ \quad + 9A_1^2 \alpha_2 k_2^2 + 9A_1^2 \alpha_3 k_3^2 + 18A_1^2 \alpha_4 k_1 k_2 - 18A_1^2 \alpha_5 k_1 k_3 - 18A_1^2 \alpha_6 k_2 k_3 - 9\varpi A_1^2 = 0, \\ \Psi^3(\xi) : 6\delta^2 \eta A_0 A_1 \alpha_1 + 6\delta^2 \eta A_0 A_1 \alpha_2 + 6\delta^2 \eta A_0 A_1 \alpha_3 + 12\delta^2 \eta A_0 A_1 \alpha_4 - 12\delta^2 \eta A_0 A_1 \alpha_5 - 12\delta^2 \eta A_0 A_1 \alpha_6 \\ \quad + 36A_0 A_1^3 \beta_3 = 0, \\ \Psi^4(\xi) : 4\delta^2 \eta A_1^2 \alpha_1 + 4\delta^2 \eta A_1^2 \alpha_2 + 4\delta^2 \eta A_1^2 \alpha_3 + 8\delta^2 \eta A_1^2 \alpha_4 - 8\delta^2 \eta A_1^2 \alpha_5 - 8\delta^2 \eta A_1^2 \alpha_6 + 9A_1^4 \beta_3 = 0. \end{array} \right.$$

We solve this system using Maple. No approximations were made; the equations are exact. We derive the solution as follow:

$$\left\{ \begin{array}{l} \varpi = \frac{(-\alpha_1 - \alpha_2 - \alpha_3 - 2\alpha_4 + 2\alpha_5 + 2\alpha_6)\delta^2}{9} + \alpha_1 k_1^2 + 2(\alpha_4 k_2 - \alpha_5 k_3) k_1 - 2\alpha_6 k_2 k_3 + \alpha_2 k_2^2 + \alpha_3 k_3^2, \\ A_0 = 0, \\ A_1 = \mp 2\delta \frac{(-\beta_3 \eta (\alpha_1 + \eta \alpha_2 + \alpha_3 + 2\alpha_4 - 2\alpha_5 - 2\alpha_6))^{1/2}}{3\beta_3} \end{array} \right. \quad (2.13)$$

By using the Eqs.(2.1), (2.4), (2.5), (2.7), (2.9) and (2.11) together, we obtain the solution of Eq. (1.1) as:

$$C(x, y, z, t) = 4^{1/3} \left(\frac{A_1 L}{4L^2 e^{\delta(x+y+z-vt)} + \eta e^{-\delta(x+y+z-vt)}} \right)^{1/3} e^{i(k_1 x + k_2 y + k_3 z + \varpi t + \phi_0)}, \quad (2.14)$$

where ϖ and A_1 are presented in Eq. (2.13) and v in Eq. (2.4). For $\eta = 4L^2$, Eq.(2.14) is equivalent to the subsequent expression, which shows the bright soliton:

$$C(x, y, z, t) = 2^{-1/3} \left(\frac{A_1}{L} \operatorname{sech}(\delta(x + y + z - vt)) \right)^{1/3} e^{i(k_1 x + k_2 y + k_3 z + \varpi t + \phi_0)}. \quad (2.15)$$

For $\eta = -4L^2$ Eq.(2.15) is equivalent to the following expression, which depicts the singular soliton:

$$C(x, y, z, t) = 2^{-1/3} \left(\frac{A_1}{L} \operatorname{csch}(\delta(x + y + z - vt)) \right)^{1/3} e^{i(k_1 x + k_2 y + k_3 z + \varpi t + \phi_0)}.$$

3. Numerical experiments and visualizations

This part of the study presents the graphical interpretation of the derived soliton, along with pertinent discussions grounded in these visualisations. Subsequent to verifying that the solutions satisfy Eq. (1.1), the subsequent logical step is to identify a suitable set of parameters to obtain a significant soliton structure. The solution function is given by Eq. (2.15), and to achieve the bright soliton representation shown in Fig.1, the ensuing parameter selection was employed: $\{L = 1.25; \eta = 4L^2; \delta = 0.75; \alpha_1 = 0.5; \alpha_2 = 0.5; \alpha_3 = 0.5; \alpha_4 = 0.5; \alpha_5 = 1; \alpha_6 = 1; k_1 = 0.25; k_2 = 0.5; k_3 = 0.5; \beta_3 = 0.55; \phi_0 = 0.5\}$. It should be emphasized that these parameter choices are not unique. Different selections can also lead to a comparable bright soliton structure, provided that the chosen parameters yield a physically meaningful solution.

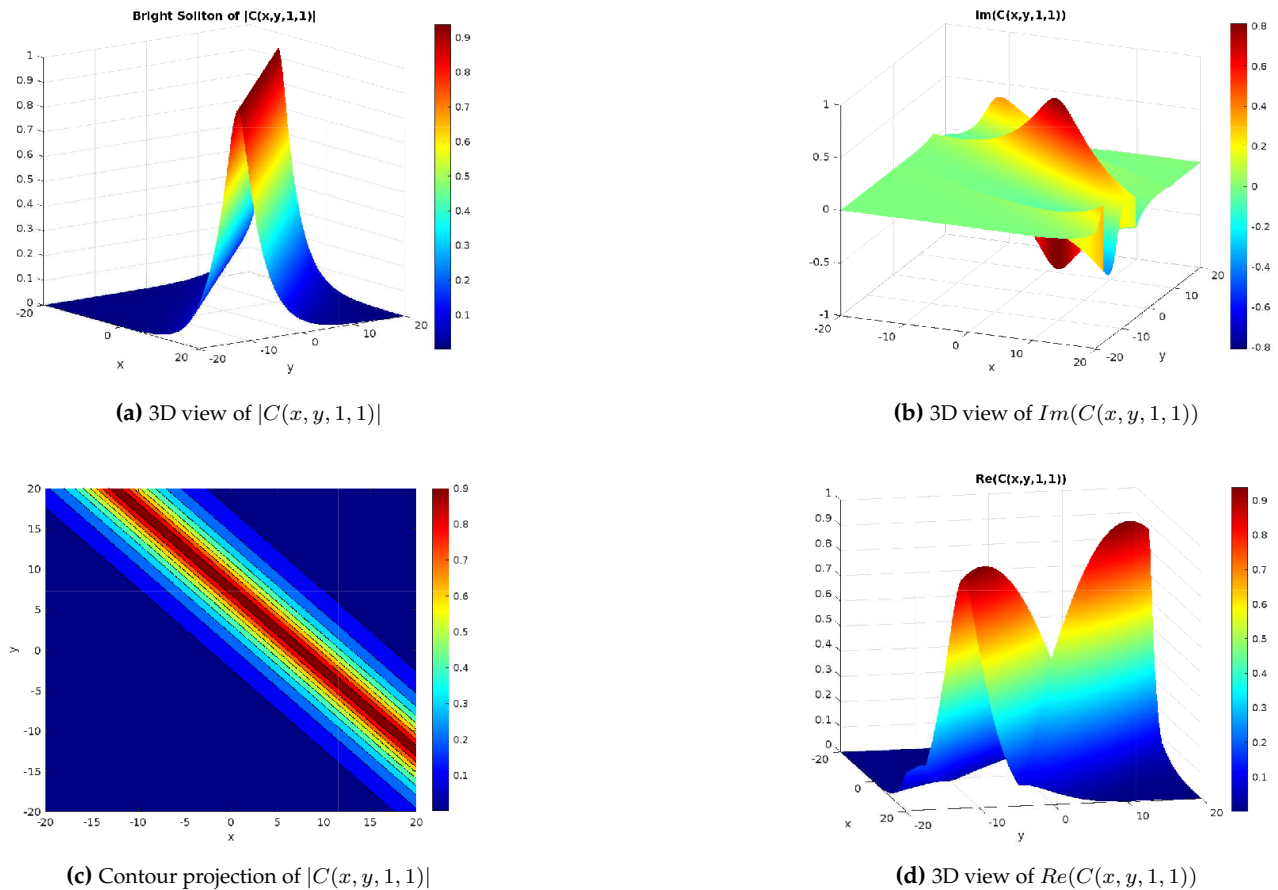
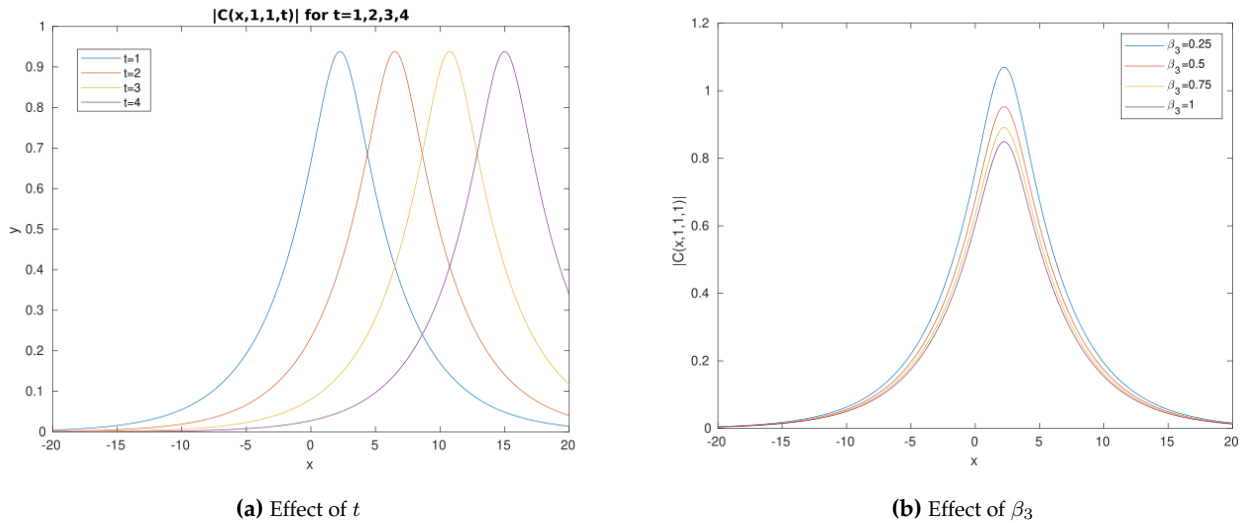


Figure 1. The visualization of Eq. (2.15)

Fig.1a illustrates a three-dimensional visualization of the function $|C(x, 1, 1, 1)|$. The plot highlights a bright soliton, where the right side exhibits a smaller flat region compared to the extended left-hand skirt along the x -axis.

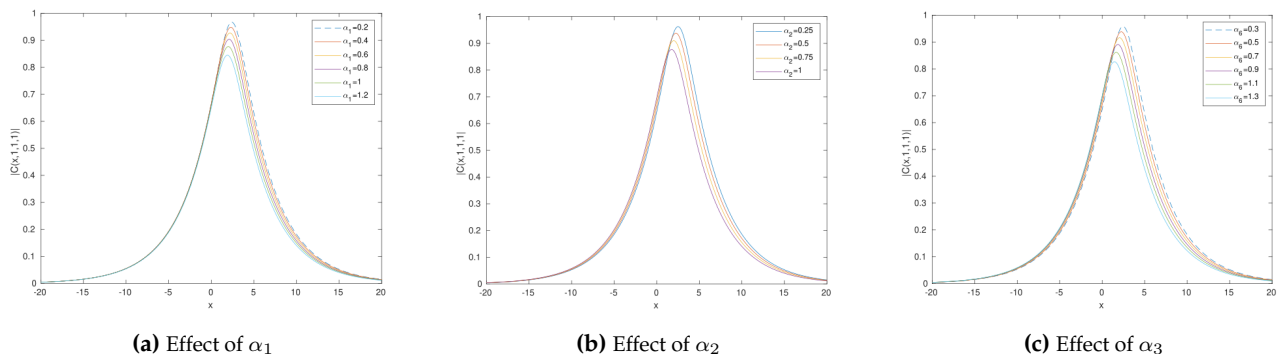
In Fig.1c, the same function is represented using a contour plot. Fig.1b presents a 3D graph of the imaginary component, $Im(C(x, y, 1, 1))$, while Fig.1d depicts the real part, $Re(C(x, y, 1, 1))$, in three dimensions.

Fig.2a illustrates the graphical depiction of $|C(x, 1, 1, t)|$, representing the bright soliton profile at various time instances $t = 1, 2, 3, 4$. The soliton propagates to the right over time while maintaining a constant amplitude. Meanwhile, Fig. 2b demonstrates how different values of β_3 ($\beta_3 = 0.25, 0.5, 0.75, 1$) influence the bright soliton. The soliton remains symmetric about a vertical axis through its peak; however, as β_3 increases, its amplitude diminishes.



(a) Effect of t (b) Effect of β_3
Figure 2. Influence of the parameters t, β_3 to the graph of Eq.(2.15)

In Fig.3 and Fig.4, the changes in the graph of the bright soliton for different values of α_i for $i = 1, \dots, 6$ are analysed. As illustrated in Fig.3 and 4, an examination of the graph depicting the variations in the bright soliton’s amplitude for distinct values of α_i for $i = 1, 2, 3, 4$ reveals a notable decrease in soliton amplitude with increasing α_i values. Additionally, a shift to the left horizontally in the soliton’s position is evident. This behaviour can be attributed to the negative coefficients associated with these parameters in Eq. (1.1). The decrease in amplitude and the shift to the left in α_4 are more pronounced than in the other parameters, due to the coefficient of α_4 in Eq. (1.1) being negative. α_5 and α_6 are the parameters with positive coefficients in Eq. (1.1), and these features are also reflected in the graph. It is evident from the graph that as the values of α_5 and α_6 increase, the amplitude of the soliton concomitantly increases and shifts to the right horizontally. The similarity in the aperture of α_5 and α_6 with α_4 can be attributed to their coefficients in the primary equation Eq. (1.1) being 2.



(a) Effect of α_1 (b) Effect of α_2 (c) Effect of α_3
Figure 3. Influence of the parameters $\alpha_1, \alpha_2, \alpha_3$

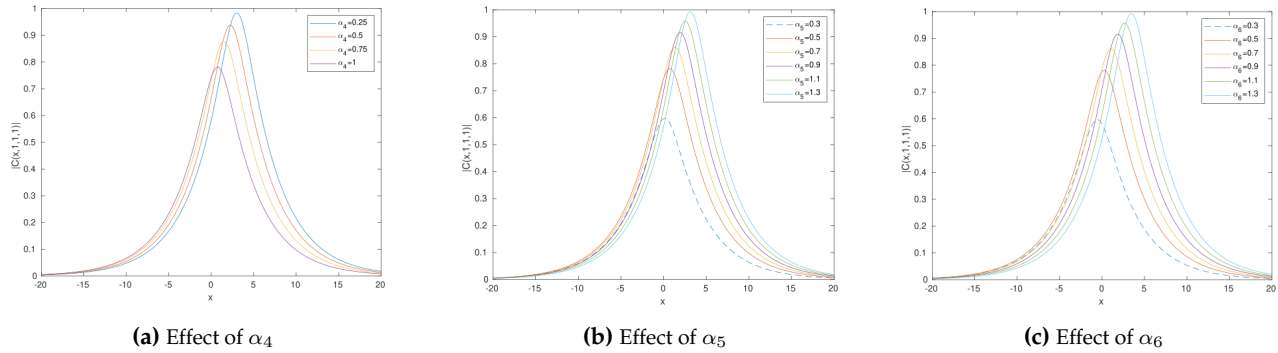


Figure 4. Influence of the parameters $\alpha_4, \alpha_5, \alpha_6$

The graphical illustrations of the obtained bright soliton solutions confirm the physical validity and localization properties of the analytical results. The solutions exhibit typical sech-type profiles -symmetric, exponentially decaying waveforms- indicating that a stable balance exists between dispersion and nonlinear self-focusing. The soliton amplitude peaks at the center and diminishes smoothly away from the core, showcasing strong spatial confinement. Moreover, the effects of dispersion and nonlinearity parameters are clearly reflected in the soliton shape: increasing the strength of the nonlinear terms, particularly the septic nonlinearity, leads to narrower and taller solitons. These observations are in agreement with established physical behaviors in nonlinear optical and plasma systems, where similar solitary structures are observed. The persistence of these solitons in a co-moving reference frame also suggests that the solutions are dynamically stable, reinforcing the applicability of the proposed model to realistic multidimensional wave propagation scenarios.

4. Conclusion

This research explores the nonlinear Schrödinger equation in $(3 + 1)$ -dimensions, incorporating cubic-quintic-septic law nonlinearity. The study focuses on transforming the equation into a nonlinear ordinary differential equation using a complex wave transformation. To achieve this, the modified Kudryashov method is implemented effectively. Consequently, optical soliton solutions are derived, particularly emphasizing the bright soliton case. The resulting bright soliton structures are visualized in three-dimensional, two-dimensional, and contour representations. Additionally, the impact of various parameters on the obtained bright soliton solutions is systematically analyzed. The derived equations are expected to benefit further research in nonlinear optics. The soliton solutions derived in this work correspond to localized wave structures that maintain their form during propagation due to a delicate balance between nonlinearity and dispersion. Physically, these bright solitons can model stable pulse propagation in nonlinear optical fibers or self-trapped states in Bose-Einstein condensates. Our graphical analysis shows how varying key parameters (e.g., nonlinear coefficients and wave numbers) affects soliton amplitude and direction of propagation. These observations have potential applications in understanding pulse shaping, filamentation, and energy localization in multidimensional media.

Article Information

Acknowledgements: The author would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Artificial Intelligence Statement: Grammar check support was received from the DeepL tool only in the Introduction section.

Conflict of Interest Disclosure: No potential conflict of interest was declared by author.

Plagiarism Statement: This article was scanned by the plagiarism program.

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