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METHODOLOGIES AND APPLICATION

A novel spherical fuzzy analytic hierarchy process and its renewable energy application

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Abstract

The extensions of ordinary fuzzy sets such as intuitionistic fuzzy sets, Pythagorean fuzzy sets, and neutrosophic sets, whose membership functions are based on three dimensions, aim at collecting experts' judgments more informatively and explicitly. In the literature, generalized three-dimensional spherical fuzzy sets have been introduced by Kutlu Gündoğdu and Kahraman (J Intell Fuzzy Syst 36(1):337–352, 2019a), including their arithmetic operations, aggregation operators, and defuzzification operations. In this paper, our aim is to extend classical analytic hierarchy process (AHP) to spherical fuzzy AHP (SF-AHP) method and to show its applicability and validity through a renewable energy location selection example and a comparative analysis between neutrosophic AHP and SF-AHP.

Keywords Spherical fuzzy sets · Multi-criteria decision making · AHP · Neutrosophic AHP

1 Introduction

Analytic hierarchy process (AHP) is one of the most popular multi-criteria decision-making methods to assess, prioritize, rank, and evaluate decision choices that was originally developed by Saaty (1980). In AHP method, factors related to a decision-making problem are categorized and consequently form a hierarchy. AHP uses the judgments of decision makers to form the decomposition of problems into hierarchies. Number of levels in the hierarchy represents problem complexity.

After the introduction of ordinary fuzzy sets by Zadeh (1965), they have been very popular in almost all branches of science. Later, various researchers have developed several extensions of ordinary fuzzy sets as illustrated in Fig. 1 with a historical order. In recent years, several researchers have utilized these extensions in the solution of multi-criteria decision-making problems. A classification of some recent publications with respect to the types of fuzzy extensions and some representative publications is as follows:

Type-2 fuzzy sets (T2FS) The concept of a type-2 fuzzy set was introduced by Zadeh (1975) as an extension of the concept of an ordinary fuzzy set called a type-1 fuzzy set. Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set (Cheng et al. 2016; Chiao 2016).

Intuitionistic fuzzy sets (IFS) Intuitionistic fuzzy sets introduced by Atanassov (1986) enable defining both the membership and non-membership degrees of an element in a fuzzy set (Wan et al. 2016; Jin et al. 2016).

Hesitant fuzzy sets (HFS) Hesitant fuzzy sets can be used as a functional tool allowing many potential degrees of membership of an element to a set. These fuzzy sets force the membership degree of an element to be possible values between zero and one (Kutlu Gündoğdu et al. 2018; Wang and Xu 2016).

Pythagorean fuzzy sets (PFS) Atanassov's intuitionistic fuzzy sets of the second type (IFS2) have been renamed by Yager (2013) as Pythagorean fuzzy sets (PFS). Hence, PFS and IFS2 mean the same fuzzy sets thereafter. IFS2 or PFS are characterized by a membership degree and a non-membership degree satisfying the condition that the square

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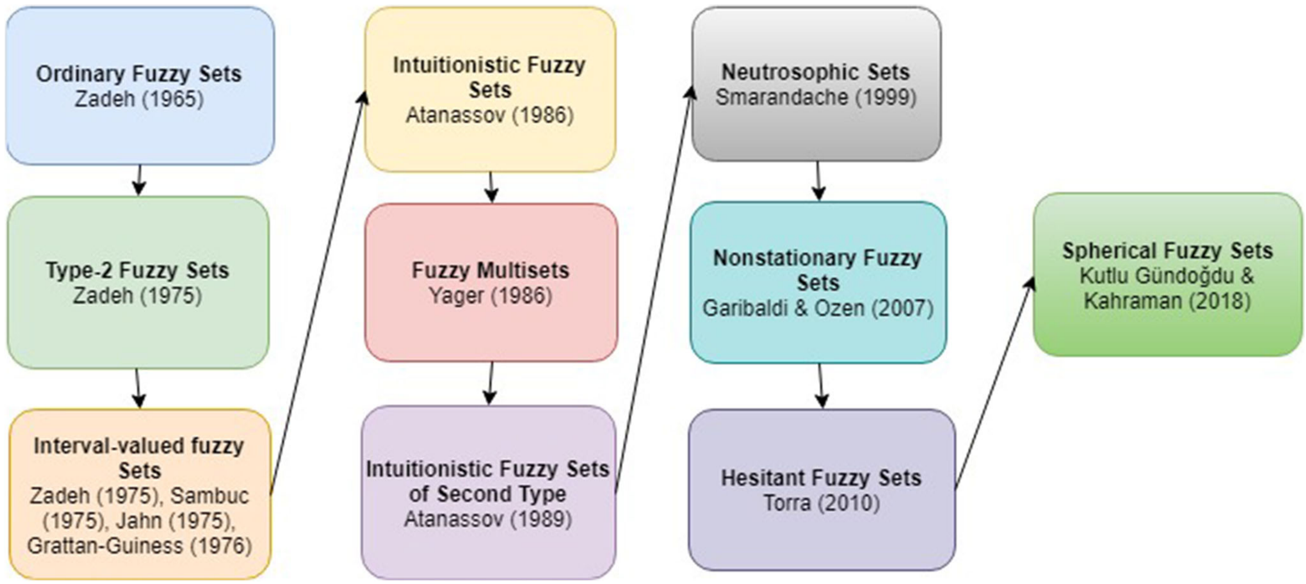


Fig. 1 Extensions of fuzzy sets

sum of its membership degree and non-membership degree is equal to or less than one, which is a generalization of intuitionistic fuzzy sets (IFS) (Liu et al. 2017a, b).

Neutrosophic sets (NS) Smarandache (1999) developed neutrosophic logic and neutrosophic sets (NSs) as an extension of intuitionistic fuzzy sets. The neutrosophic set is defined as the set where each element of the universe has a degree of truthiness, indeterminacy, and falsity (Smarandache 2003).

In this paper, we first develop a novel spherical fuzzy AHP method and then apply it to renewable energy site selection problem. Fossil energy sources are running out due to the high increase in energy consumption. There are a lot of ways to obtain energy without fossil fuels, which are called renewable energy sources (RES). Environmental problems caused by fossil energy sources can be removed through these renewable energy sources. Using more RES can be considered as one of the most powerful solutions to address the environmental problems. The evaluation of renewable energy alternatives requires several linguistic criteria to be included in the decision process. The numerical definitions of these criteria are realized by spherical fuzzy sets incorporating a new point of view to decision making under fuzziness. The independent assignment of membership parameters with larger domains brings a novelty to the evaluation process of renewable energy location alternatives.

Kutlu Gündoğdu and Kahraman (2019a, b, c) have recently introduced the spherical fuzzy sets (SFS). These sets are based on the fact that the hesitancy of a decision maker can be defined independently from membership and non-membership degrees, satisfying the following condition (Kutlu Gündoğdu and Kahraman 2019a):

$$0 \leq \mu_{\tilde{A}}^2(u) + v_{\tilde{A}}^2(u) + \pi_{\tilde{A}}^2(u) \leq 1 \quad \forall u \in U \tag{1}$$

where $\mu_{\tilde{A}}(u)$, $v_{\tilde{A}}(u)$ and $\pi_{\tilde{A}}(u)$ are the degrees of membership, non-membership, and hesitancy of u to $\tilde{A}$ for each u, respectively.

On the surface of the sphere, Eq. (1) becomes

$$\mu_{\tilde{A}}^2(u) + v_{\tilde{A}}^2(u) + \pi_{\tilde{A}}^2(u) = 1 \qquad \forall u \in U \tag{2}$$

The idea behind SFS is to let decision makers to generalize other extensions of fuzzy sets by defining a membership function on a spherical surface and independently assign the parameters of that membership function with a larger domain. SFS are a synthesis of PFS and NS. Spherical fuzzy TOPSIS, spherical fuzzy WASPAS, and spherical fuzzy VIKOR methods have been developed by Kutlu Gündoğdu and Kahraman (2019a, b, c, d).

In this study, a decision-making model for site selection of wind power farm is developed based on spherical fuzzy sets as the expert evaluations of location alternatives involve uncertainty and vagueness. This decision model integrates analytic hierarchy process (AHP) with spherical fuzzy sets. A case study containing four criteria, three subcriteria under each criterion, and four alternatives is presented.

The originality of the paper comes from its presentation of a novel SF-AHP and the application of the proposed method in the renewable energy industry. The SF-AHP enables decision makers to independently reflect their hesitancies in the decision process by using a linguistic evaluation scale based on spherical fuzzy sets.

The rest of this paper is organized as follows. Section 2 includes the introductory definitions and the preliminaries on SFS. Section 3 summarizes a literature review on Fuzzy AHP. Section 4 includes our proposed MCDM technique, spherical fuzzy AHP method (SF-AHP). Section 5 applies SF-AHP method to a wind power site selection problem and includes a comparative analysis of SF-AHP and NS-AHP. Finally, the study is concluded in the last section.

2 Spherical fuzzy sets: preliminaries

Intuitionistic and Pythagorean fuzzy membership functions are composed of membership, non-membership, and hesitancy parameters, which can be calculated by $\pi_{\tilde{I}} = 1 - \mu_{\tilde{I}} - v_{\tilde{I}}$ or $\pi_{\tilde{P}} = (1 - \mu_{\tilde{P}}^2(u) - v_{\tilde{P}}^2(u))^{1/2}$, respectively. Neutrosophic membership functions are also defined by three parameters *truthiness, falsity*, and *indeterminacy,* whose sum can be between 0 and 3, and the value of each is between 0 and 1 independently. In spherical fuzzy sets, while the squared sum of *membership*, non-membership, and hesitancy parameters can be between 0 and 1, each of them can be defined between 0 and 1 independently to satisfy that their squared sum is at most equal to 1. Figure 2 illustrates the differences among IFS, PFS, NS, and SFS.

In this section, we give the definition of SFS and summarize spherical distance measurement, arithmetic operations, aggregation operators, and defuzzification operations.

Definition 1 (*Spherical fuzzy sets (SFS)* $\tilde{A}_S$) Let U_1 and U_2 be two universes. Let two spherical fuzzy sets $\tilde{A}_S$ and $\tilde{B}_S$ of the universe of discourse U_1 and U_2 be as follows:

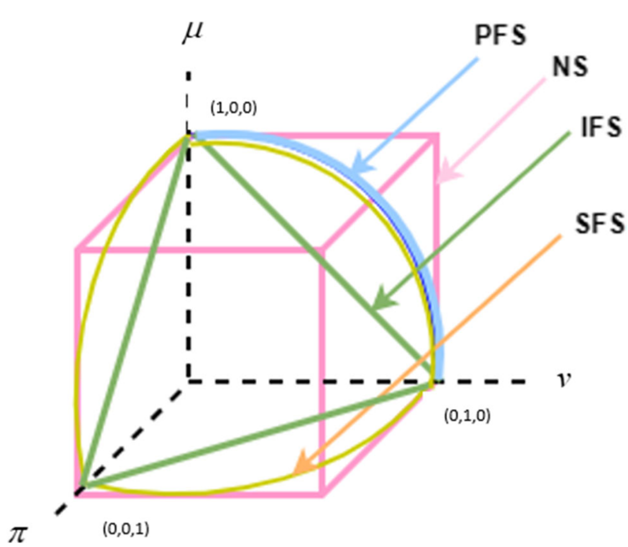


Fig. 2 Geometric representations of IFS, PFS, NS, and SFS

$$\tilde{A}_S = \left\{x, \left(\mu_{\tilde{A}_S}(x), v_{\tilde{A}_S}(x), \pi_{\tilde{A}_S}(x)\right) \middle| x \in U_1\right\} \tag{3}$$

where

$$\mu_{\tilde{A}_S}(x) : U_1 \to [0,1], \quad v_{\tilde{A}_S}(x) : U_1 \to [0,1], \quad \pi_{\tilde{A}_S}(x) : U_1 \to [0,1]$$

and

$$0 \le \mu_{\tilde{A}_S}^2(x) + v_{\tilde{A}_S}^2(x) + \pi_{\tilde{A}_S}^2(x) \le 1 \qquad \forall x \in U_1 \tag{4}$$

For each x, the $\mu_{\tilde{A}_S}(x)$, $v_{\tilde{A}_S}(x)$ and $\pi_{\tilde{A}_S}(x)$ are the degrees of membership, non-membership, and hesitancy of x to $\tilde{A}_S$, respectively.

$$\tilde{B}_S = \left\{y, \left(\mu_{\tilde{B}_S}(y), v_{\tilde{B}_S}(y), \pi_{\tilde{B}_S}(y)\right) \middle| y \in U_2\right\} \tag{5}$$

where

$$\mu_{\tilde{B}_S}(y) : U_2 \to [0,1], \quad v_{\tilde{B}_S}(y) : U_2 \to [0,1], \quad \pi_{\tilde{B}_S}(y) : U_2 \to [0,1]$$

and

$$0 \le \mu_{\tilde{B}_S}^2(y) + v_{\tilde{B}_S}^2(y) + \pi_{\tilde{B}_S}^2(y) \le 1 \qquad \forall y \in U_2 \tag{6}$$

For each y, the numbers $\mu_{\tilde{B}_S}(y)$, $v_{\tilde{B}_S}(y)$ and $\pi_{\tilde{B}_S}(y)$ are the degrees of membership, non-membership, and hesitancy of y to $\tilde{B}_S$, respectively (Kutlu Gündoğdu and Kahraman 2019a).

Zadeh's extension principle extends the classical arithmetic operations to their fuzzy correspondings. In the following, we defined the extension principle for single-valued spherical fuzzy sets.

Proposition 1 *The following Cartesian product of SFS is considered:*

$$\tilde{A}_S \times_2 \tilde{B}_S = \left\{\left((x,y), \min\left(\mu_{\tilde{A}_S}(x), \mu_{\tilde{B}_S}(y)\right), \max\left(v_{\tilde{A}_S}(x), v_{\tilde{B}_S}(y)\right), \min\left(\pi_{\tilde{A}_S}(x), \pi_{\tilde{B}_S}(y)\right)\right) \middle| x \in U_1, y \in U_2\right\} \tag{7}$$

Let for $i = 1, \ldots, n, U_i$ *be a universe and* $\tilde{A}_i = \left\{\left\langle x, (\mu_{\tilde{A}_i}(x), v_{\tilde{A}_i}(x), \pi_{\tilde{A}_i}(x)) \middle| x \in U_i\right\}\right.$ *be a SFS. Then, Cartesian product of SFS*:

$$\tilde{B}_s^n = \mathop{X}_{i=1}^{n} \tilde{A}_{si} = \left\{\left((x_1, x_2, \ldots x_n), \min_{i=1}^{n} \mu_{\tilde{A}_{si}}(x_i), \max_{i=1}^{n} v_{\tilde{A}_{si}}(x_i), \min_{i=1}^{n} \pi_{\tilde{A}_{si}}(x_i)\right) \middle| \forall x_i \in U_i, i = 1, \ldots, n\right\}$$ is a SFS on is $\mathop{X}_{i=1}^{n} U_i$.

Proof We prove by inductive reasoning. For $n = 2$, the result is given in Eq. (5). By inductive reasoning, $B^{n-1} = X_{i=1}^{n-1} A_i$ is a SFS on $X_{i=1}^{n-1} U_i$, and hence, $B^n = B^{n-1} \times_2 \tilde{A}_n = X_{i=1}^{n-1} A_i$ is a SFS on $X_{i=1}^{n-1} U_i$.

Proposition 2 *Zadeh's Extension Principle for SFS. Let for $i = 1, \ldots, n$, U_i be a universe and let $V \neq \emptyset$. Let $f : X_{i=1}^{n-1} U_i \rightarrow V$ be a mapping, where $y = f(z_1, \ldots, z_n)$. Let z_i be a linguistic variable on U_i for $i = 1, \ldots, n$. Assume that for all i, $\tilde{A}_{si}$ is a SFS on U_i,* and *then, the output of mapping f is $\tilde{B}_S$. For $y \in V$, the set $\tilde{B}_S$ is a SFS on V defined as follows*:

$$\tilde{B}_s(y) = \left\{ \left(\max_{Z(y)} \left(\min_{i=1}^{n} \mu_{\tilde{A}_{si}}(u_i) \right), \min_{Z(y)} \left(\max_{i=1}^{n} v_{\tilde{A}_{si}}(u_i) \right), \min_{Z(y)} \left(\min_{i=1}^{n} \pi_{\tilde{A}_{si}}(u_i) \right) \right) \middle| \forall u_i \in U_i, i = 1, \ldots, n \right\}, \text{ if } f^{-1}(y) \neq \emptyset$$

where $Z(y) = f^{-1}(y)$

For the addition and multiplication operators,

$$\tilde{A}_S \oplus \tilde{B}_S = \left\{ z, \left(\max_{z=x+y} \min\left\{ \mu_{\tilde{A}_S}(x), \mu_{\tilde{B}_S}(y) \right\} \right), \left(\min_{z=x+y} \max\left\{ v_{\tilde{A}_S}(x), v_{\tilde{B}_S}(y) \right\} \right), \left(\min_{z=x+y} \min\left\{ \pi_{\tilde{A}_S}(x), \pi_{\tilde{B}_S}(y) \right\} \right) \right\} \tag{8}$$

$$\tilde{A}_S \otimes \tilde{B}_S = \left\{ z, \left(\max_{z=x*y} \min\left\{ \mu_{\tilde{A}_S}(x), \mu_{\tilde{B}_S}(y) \right\} \right), \left(\min_{z=x*y} \max\left\{ v_{\tilde{A}_S}(x), v_{\tilde{B}_S}(y) \right\} \right), \left(\min_{z=x*y} \min\left\{ \pi_{\tilde{A}_S}(x), \pi_{\tilde{B}_S}(y) \right\} \right) \right\} \tag{9}$$

On the basis of relationship between SFS and PFS, Kutlu Gündoğdu and Kahraman (2019a) further define some novel operations for SFS as below:

Definition 2 Basic operators (Kutlu Gündoğdu and Kahraman 2019a)

Union

$$\tilde{A}_S \cup \tilde{B}_S = \left\{ \max\{\mu_{\tilde{A}_S}, \mu_{\tilde{B}_S}\}, \min\{v_{\tilde{A}_S}, v_{\tilde{B}_S}\}, \min\left\{ \left(1 - \left(\left(\max\left\{ \mu_{\tilde{A}_S}, \mu_{\tilde{B}_S} \right\} \right)^2 + \left(\min\left\{ v_{\tilde{A}_S}, v_{\tilde{B}_S} \right\} \right)^2 \right) \right)^{1/2}, \max\left\{ \pi_{\tilde{A}_S}, \pi_{\tilde{B}_S} \right\} \right\} \right\} \tag{10}$$

Intersection

$$\tilde{A}_S \cap \tilde{B}_S = \left\{ \min\{\mu_{\tilde{A}_S}, \mu_{\tilde{B}_S}\}, \max\{v_{\tilde{A}_S}, v_{\tilde{B}_S}\}, \max\left\{ \left(1 - \left(\left(\min\left\{ \mu_{\tilde{A}_S}, \mu_{\tilde{B}_S} \right\} \right)^2 + \left(\max\left\{ v_{\tilde{A}_S}, v_{\tilde{B}_S} \right\} \right)^2 \right) \right)^{1/2}, \min\left\{ \pi_{\tilde{A}_S}, \pi_{\tilde{B}_S} \right\} \right\} \right\} \tag{11}$$

Addition

$$\tilde{A}_S \oplus \tilde{B}_S = \left\{ \left(\mu_{\tilde{A}_S}^2 + \mu_{\tilde{B}_S}^2 - \mu_{\tilde{A}_S}^2 \mu_{\tilde{B}_S}^2 \right)^{1/2}, v_{\tilde{A}_S} v_{\tilde{B}_S}, \left(\left(1 - \mu_{\tilde{B}_S}^2 \right) \pi_{\tilde{A}_S}^2 + \left(1 - \mu_{\tilde{A}_S}^2 \right) \pi_{\tilde{B}_S}^2 - \pi_{\tilde{A}_S}^2 \pi_{\tilde{B}_S}^2 \right)^{1/2} \right\} \tag{12}$$

Multiplication

$$\tilde{A}_S \otimes \tilde{B}_S = \left\{ \mu_{\tilde{A}_S} \mu_{\tilde{B}_S}, \left(v_{\tilde{A}_S}^2 + v_{\tilde{B}_S}^2 - v_{\tilde{A}_S}^2 v_{\tilde{B}_S}^2 \right)^{1/2}, \left(\left(1 - v_{\tilde{B}_S}^2 \right) \pi_{\tilde{A}_S}^2 + \left(1 - v_{\tilde{A}_S}^2 \right) \pi_{\tilde{B}_S}^2 - \pi_{\tilde{A}_S}^2 \pi_{\tilde{B}_S}^2 \right)^{1/2} \right\} \tag{13}$$

Multiplication by a scalar; $\lambda > 0$

$$\lambda \cdot \tilde{A}_S = \left\{ \left(1 - \left(1 - \mu_{\tilde{A}_S}^2 \right)^{\lambda} \right)^{1/2}, v_{\tilde{A}_S}^{\lambda}, \left(\left(1 - \mu_{\tilde{A}_S}^2 \right)^{\lambda} - \left(1 - \mu_{\tilde{A}_S}^2 - \pi_{\tilde{A}_S}^2 \right)^{\lambda} \right)^{1/2} \right\} \tag{14}$$

Power of $\tilde{A}_S$; $\lambda > 0$

$$\tilde{A}_S^{\lambda} = \left\{ \mu_{\tilde{A}_S}^{\lambda}, \left(1 - \left(1 - v_{\tilde{A}_S}^2 \right)^{\lambda} \right)^{1/2}, \left(\left(1 - v_{\tilde{A}_S}^2 \right)^{\lambda} - \left(1 - v_{\tilde{A}_S}^2 - \pi_{\tilde{A}_S}^2 \right)^{\lambda} \right)^{1/2} \right\} \tag{15}$$

Definition 3 For these SFS $\tilde{A}_S = (\mu_{\tilde{A}_S}, v_{\tilde{A}_S}, \pi_{\tilde{A}_S})$ and $\tilde{B}_S = (\mu_{\tilde{B}_S}, v_{\tilde{B}_S}, \pi_{\tilde{B}_S})$, the followings are valid under the condition $\lambda, \lambda_1, \lambda_2 > 0$ (Kutlu Gündoğdu and Kahraman 2019a, b, c, d).

i. $\tilde{A}_S \oplus \tilde{B}_S = \tilde{B}_S \oplus \tilde{A}_S$ (16)

ii. $\tilde{A}_S \otimes \tilde{B}_S = \tilde{B}_S \otimes \tilde{A}_S$ (17)

iii. $\lambda(\tilde{A}_S \oplus \tilde{B}_S) = \lambda\tilde{A}_S \oplus \lambda\tilde{B}_S$ (18)

iv. $\lambda_1\tilde{A}_S \oplus \lambda_2\tilde{A}_S = (\lambda_1 + \lambda_2)\tilde{A}_S$ (19)

v. $(\tilde{A}_S \otimes \tilde{B}_S)^{\lambda} = \tilde{A}_S^{\lambda} \otimes \tilde{B}_S^{\lambda}$ (20)

vi. $\tilde{A}_S^{\lambda_1} \otimes \tilde{A}_S^{\lambda_2} = \tilde{A}_S^{\lambda_1 + \lambda_2}$ (21)

Definition 4 Spherical weighted arithmetic mean (SWAM) with respect to, $w = (w_1, w_2 \ldots, w_n)$; $w_i \in [0, 1]$; $\sum_{i=1}^{n} w_i = 1$, SWAM is defined as:

$$SWAM_w(\tilde{A}_{S1},\ldots,\tilde{A}_{Sn}) = w_1\tilde{A}_{S1} + w_2\tilde{A}_{S2} + \cdots + w_n\tilde{A}_{Sn} = \left\{ \left[1 - \prod_{i=1}^{n}(1-\mu^2_{\tilde{A}_{Si}})^{w_i}\right]^{1/2}, \prod_{i=1}^{n} v^{w_i}_{\tilde{A}_{Si}}, \left[\prod_{i=1}^{n}(1-\mu^2_{\tilde{A}_{Si}})^{w_i} - \prod_{i=1}^{n}(1-\mu^2_{\tilde{A}_{Si}} - \pi^2_{\tilde{A}_{Si}})^{w_i}\right]^{1/2} \right\} \quad (22)$$

Definition 5 Spherical weighted geometric mean (SWGM) with respect to, $w = (w_1, w_2\ldots, w_n)$; $w_i \in [0,1]$; $\sum_{i=1}^{n} w_i = 1$, SWGM is defined as:

$$SWGM_w(\tilde{A}_{S1},\ldots,\ddot{A}_{Sn}) = \tilde{A}^{w_1}_{S1} + \tilde{A}^{w_2}_{S2} + \cdots + \tilde{A}^{w_n}_{Sn} = \left\{ \prod_{i=1}^{n} \mu^{w_i}_{\tilde{A}_{Si}}, \left[1 - \prod_{i=1}^{n}(1-v^2_{\tilde{A}_{Si}})^{w_i}\right]^{1/2}, \left[\prod_{i=1}^{n}(1-v^2_{\tilde{A}_{Si}})^{w_i} - \prod_{i=1}^{n}(1-v^2_{\tilde{A}_{Si}} - \pi^2_{\tilde{A}_{Si}})^{w_i}\right]^{1/2} \right\} \quad (23)$$

3 A literature review on fuzzy AHP

One of the most popular MCDM methods, analytic hierarchy process (AHP), is introduced by Saaty (1980) as a structured approach used for decision making in complex problems. AHP method aims quantifying relative priorities for a given set of alternatives based on the decision-makers' pairwise judgments. This method allows constructing the decision-making criteria as a hierarchy, calculating the weights of the criteria and alternatives, and it also stresses the consistency of the comparison of alternatives.

In the classical method, decision-makers' evaluations are represented as crisp numbers. However, in cases, where decision makers cannot express the assessments by crisp numbers, fuzzy logic can be used which provides a mathematical strength to capture the uncertainties accompanying with human cognitive process (Kahraman and Kaya 2010). Hence, original AHP method has been extended to several fuzzy versions due to incomplete information and uncertainty. The first algorithm in fuzzy AHP by using triangular fuzzy membership functions is proposed by Van Laarhoven and Pedrycz (1983). AHP with trapezoidal fuzzy numbers and the geometric mean method to derive fuzzy weights and performance scores are developed by Buckley (1985). Chang (1996) proposes using extent analysis method for the synthetic extent values of the pairwise comparisons by utilizing triangular fuzzy numbers. In one of the recent studies Zeng et al. (2007) develop using arithmetic averaging method to get performance scores and extend the method with different scales include triangular, trapezoidal, and crisp numbers. Some of other extensions are ordinary fuzzy AHP with type-1 fuzzy sets (Tan et al. 2014), fuzzy AHP with type 2 fuzzy sets (Kahraman et al. 2014; Oztaysi et al. 2017), intuitionistic fuzzy AHP with intuitionistic fuzzy sets (Wu et al. 2013), fuzzy AHP with hesitant fuzzy sets (Öztaysi et al. 2015; Boltürk et al. 2016), fuzzy AHP with interval-valued intuitionistic sets (Tooranloo and Iranpour 2017), neutrosophic AHP method (Abdel-Basset et al. 2018; Bolturk and Kahraman 2018), and Pythagorean fuzzy AHP method (Ilbahar et al. 2018). Fuzzy AHP method has been also integrated with several other approaches in the literature

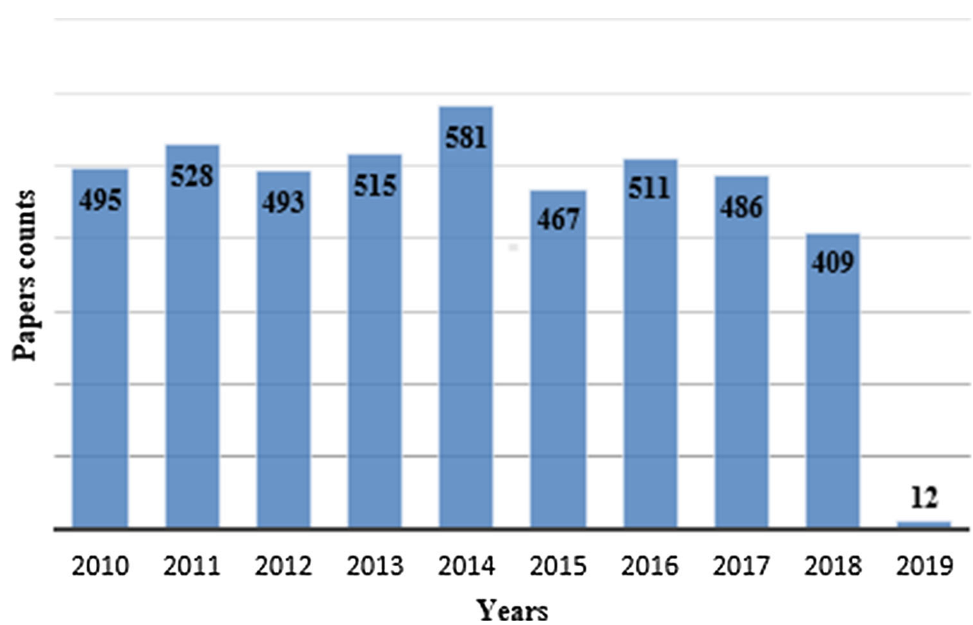


Fig. 3 Fuzzy AHP studies based on years 2010–2018

(Rezaei et al. 2014; Ozgen and Gulsun 2014; Gim and Kim 2014; Kaya et al. 2012).

Group decision making in fuzzy AHP when there exist more than one decision makers has been another attractive research area in the literature. The models in this area can be classified as follows: the models dealing with incomplete information (Capuano et al. 2017), the models based on consensus measures (Cabrerizo et al. 2017), and the models based on heterogeneous preference relations (Liu et al. 2017a, b). These models present some advantages when compared with other decision models.

A literature review on fuzzy AHP using SCOPUS gives 4497 published papers in all fields. Among these, 3331 papers mention fuzzy AHP in "article title, abstract, or keywords" and 1166 papers in their titles. Yearly distribution of papers using fuzzy AHP is given in Fig. 3.

Figure 4 indicates number of papers published on fuzzy AHP up to twelve authors. Authors C. Kahraman (with 51 publications) from Istanbul Technical University, E. K: Zavadskas (with 20 publications) from Vilnius Gediminas Technical University, M. K. Barua (with 13 publications) from Indian Institute of Technology Roorkee and G. Buyukozkan (with 12 publications) from Galatasaray University are the most productive researchers in this field.

Fuzzy AHP method has been used in different areas. These areas can be categorized as follows: engineering, computer science, business management, mathematics, environmental science, decision sciences, social sciences, energy, earth and planetary sciences, agricultural and biological sciences, and other areas as represented in Fig. 5. Particularly, in the engineering and computer science areas, the method has been extensively used.

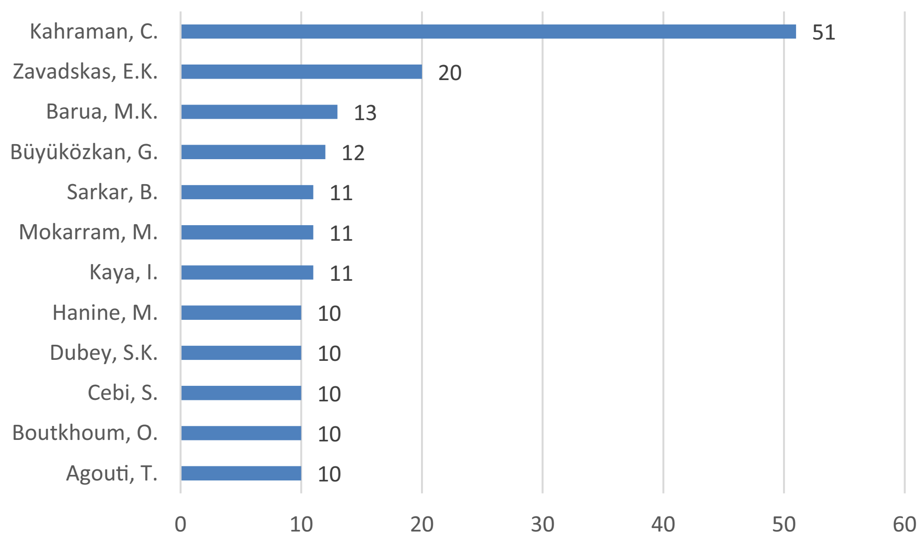
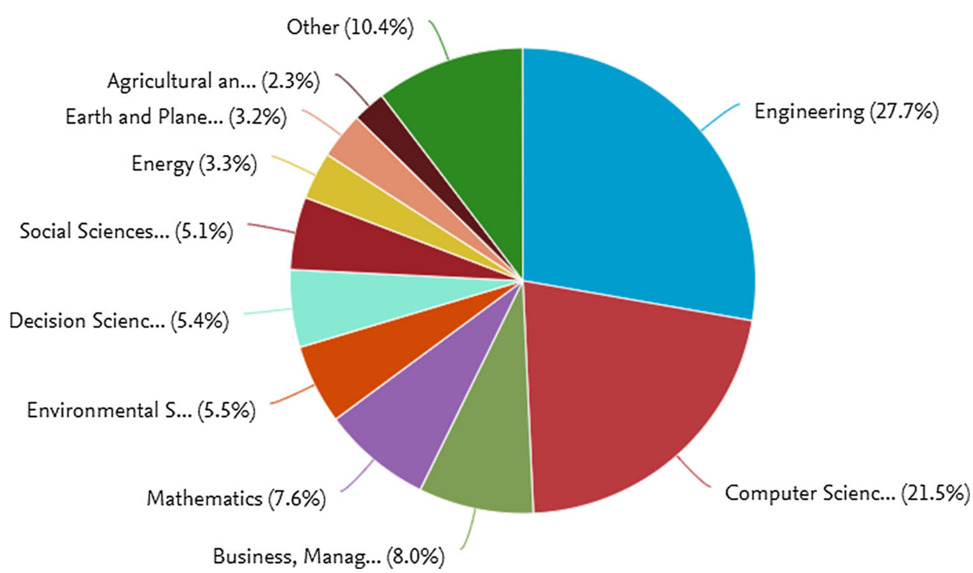


Fig. 4 Fuzzy AHP publications with respect to authors

Fig. 5 Percentages of fuzzy AHP studies based on areas



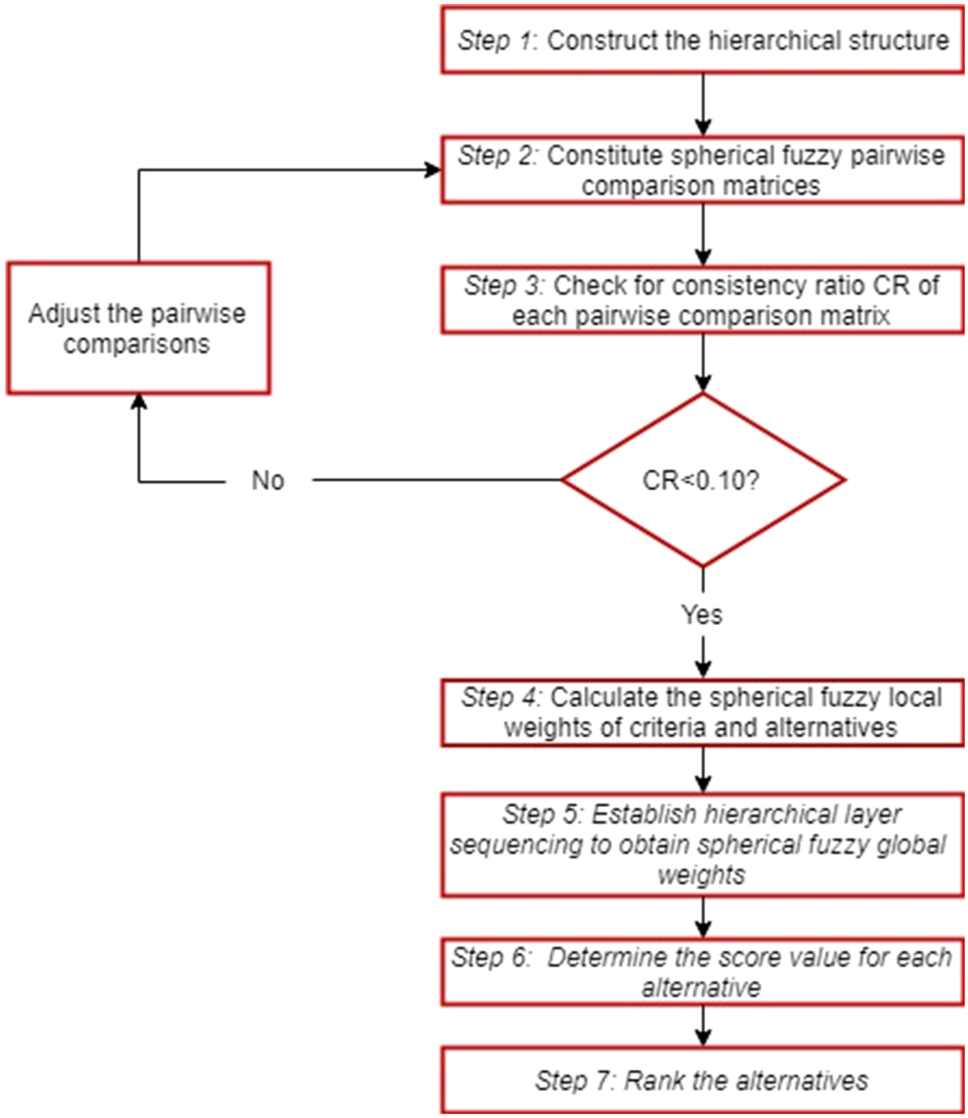


Fig. 6 Flowchart of proposed SF-AHP method

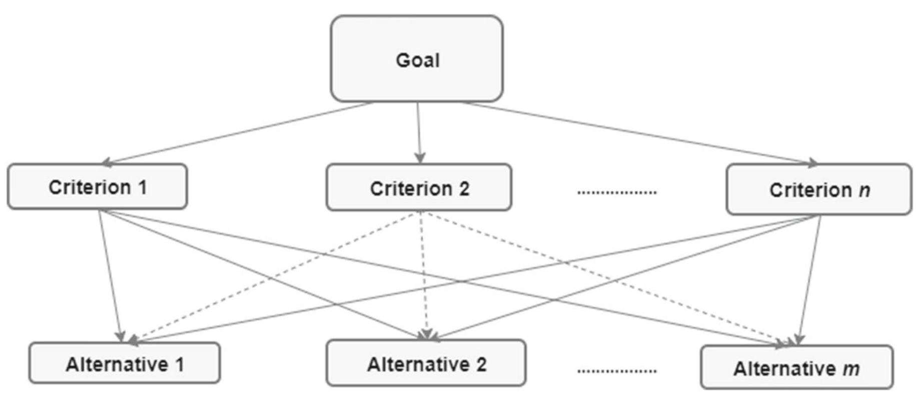


Fig. 7 A hierarchical structure

Table 1 Linguistic measures of importance used for pairwise comparisons

	(μ, v, π)	Score Index (SI)
Absolutely more importance (AMI)	(0.9, 0.1, 0.0)	9
Very high importance (VHI)	(0.8, 0.2, 0.1)	7
High importance (HI)	(0.7, 0.3, 0.2)	5
Slightly more importance (SMI)	(0.6, 0.4, 0.3)	3
Equally importance (EI)	(0.5, 0.4, 0.4)	1
Slightly low importance (SLI)	(0.4, 0.6, 0.3)	1/3
Low importance (LI)	(0.3, 0.7, 0.2)	1/5
Very low importance (VLI)	(0.2, 0.8, 0.1)	1/7
Absolutely low importance (ALI)	(0.1, 0.9, 0.0)	1/9

Table 2 Pairwise comparison of main criteria

Criteria	C1	C2	C3	C4	$\tilde{w}^s$	$\bar{w}^s$
C1	EI	HI	AMI	SMI	(0.73, 0.26, 0.23)	0.360
C2	LI	EI	SMI	LI	(0.35, 0.63, 0.27)	0.158
C3	ALI	SLI	EI	VLI	(0.37, 0.61, 0.29)	0.169
C4	SLI	HI	VHI	EI	(0.65, 0.35, 0.25)	0.313

CR = 0.066

4 Extension of AHP with spherical fuzzy sets

The proposed spherical fuzzy AHP method is composed of several steps as given in this section. Before giving these steps, we present the flowchart of the SF-AHP method in Fig. 6 in order to make it easily understandable.

Step 1 Construct the hierarchical structure

In this step, a hierarchical structure consisting of at least three levels is developed (Fig. 7). Level 1 represents a goal or an objective (selecting the best alternative) based on score index. The score index is estimated based on a finite set of criteria $C = \{C_1, C_2, \ldots C_n\}$, which are shown at Level 2. There are many sub-criteria defined for any criterion C in this hierarchical structure. Therefore, at Level 3, a discrete set of m feasible alternative $X = \{x_1, x_2, \ldots x_m\}$ $(m \geq 2)$ is defined.

Step 2 Constitute pairwise comparisons using spherical fuzzy judgment matrices based on the linguistic terms given in Table 1.

Equations (24) and (25) are used to obtain the score indices (SI) in Table 2.

$$\mathrm{SI} = \sqrt{\left|100 * \left[\left(\mu_{\tilde{A}_s} - \pi_{\tilde{A}_s}\right)^2 - \left(v_{\tilde{A}_s} - \pi_{\tilde{A}_s}\right)^2\right]\right|} \tag{24}$$

for AMI, VHI, HI, SMI, and EI

$$\frac{1}{\mathrm{SI}} = \frac{1}{\sqrt{\left|100 * \left[\left(\mu_{\tilde{A}_s} - \pi_{\tilde{A}_s}\right)^2 - \left(v_{\tilde{A}_s} - \pi_{\tilde{A}_s}\right)^2\right]\right|}} \tag{25}$$

for EI, SLI, LI, VLI, and ALI

Step 3 Check for the consistency of each pairwise comparison matrix (J). To do that, convert the linguistic terms in the pairwise comparison matrix to their corresponding score indices. Then, apply the classical consistency check. The threshold of the CR is 10%. For instance, the pairwise comparison matrix $J = \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix}\begin{vmatrix} \mathrm{EI} & \mathrm{SLI} & \mathrm{SMI} \\ \mathrm{SMI} & \mathrm{EI} & \mathrm{HI} \\ \mathrm{SLI} & \mathrm{LI} & \mathrm{EI} \end{vmatrix}$ is converted to $J = \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix}\begin{vmatrix} 1 & 1/3 & 3 \\ 3 & 1 & 5 \\ 1/3 & 1/5 & 1 \end{vmatrix}$ and the consistency ratio is calculated by using the classical way and found to be 0.03457, which indicates that the pairwise comparison matrix is consistent.

Step 4 Calculate the spherical fuzzy local weights of criteria and alternatives.

Determine the weight of each alternative using SWAM operator given in Eq. (26) with respect to each criterion. The weighted arithmetic mean is used to compute the spherical fuzzy weights.

$$\begin{aligned} \mathrm{SWAM}_w(A_{S1}, \ldots, A_{Sn}) &= w_1 A_{S1} + w_2 A_{S2} + \cdots + w_n A_{Sn} \\ &= \left\langle \left[1 - \prod_{i=1}^{n} (1 - \mu_{A_{Si}}^2)^{w_i}\right]^{1/2}, \prod_{i=1}^{n} v_{A_{Si}}^{w_i}, \right. \\ &\quad \left. \left[\prod_{i=1}^{n} (1 - \mu_{A_{Si}}^2)^{w_i} - \prod_{i=1}^{n} (1 - \mu_{A_{Si}}^2 - \pi_{A_{Si}}^2)^{w_i}\right]^{1/2} \right\rangle \end{aligned} \tag{26}$$

where $w = 1/n$.

Step 5 Establish the hierarchical layer sequencing to obtain global weights.

The spherical fuzzy weights at each level are aggregated to estimate final ranking orders for the alternatives. This computation is carried out from bottom level (alternatives) to top level (goal) as shown in Fig. 4.

At this point, there are two possible ways to follow. The first one is to defuzzify the criteria weights by using the score function (S) in Eq. (27) and then normalize them by Eq. (28) and apply spherical fuzzy multiplication given in Eq. (29).

$$S(\tilde{w}_j^s) = \sqrt{\left|100 * \left[\left(3\mu_{\tilde{A}_s} - \frac{\pi_{\tilde{A}_s}}{2}\right)^2 - \left(\frac{v_{\tilde{A}_s}}{2} - \pi_{\tilde{A}_s}\right)^2\right]\right|} \quad (27)$$

Normalize the criteria weights by using Eq. (28).

$$\bar{w}_j^s = \frac{S(\tilde{w}_j^s)}{\sum_{J=1}^n S(\tilde{w}_j^s)} \quad (28)$$

$$\tilde{A}_{S_{ij}} = \bar{w}_j^s \cdot \tilde{A}_{S_i} = \left\langle \left(1 - \left(1 - \mu_{\tilde{A}_S}^2\right)^{\bar{w}_j^s}\right)^{1/2}, v_{\tilde{A}_S}^{\bar{w}_j^s}, \left(\left(1 - \mu_{\tilde{A}_S}^2\right)^{\bar{w}_j^s} - \left(1 - \mu_{\tilde{A}_S}^2 - \pi_{\tilde{A}_S}^2\right)^{\bar{w}_j^s}\right)^{1/2} \right\rangle \quad \forall i \quad (29)$$

The final spherical fuzzy AHP score ($\tilde{F}$), for each alternative A_i, is obtained by carrying out the spherical fuzzy arithmetic addition over each global preference weights as given in Eq. (30).

$$\tilde{F} = \sum_{j=1}^{n} \tilde{A}_{S_{ij}} = \tilde{A}_{S_{i1}} \oplus \tilde{A}_{S_{i2}} \cdots \oplus \tilde{A}_{S_{in}} \quad \forall i$$
$$\text{i.e. } \tilde{A}_{S_{11}} \oplus \tilde{A}_{S_{12}} = \left\langle \left(\mu_{\tilde{A}_{S_{11}}}^2 + \mu_{\tilde{A}_{S_{12}}}^2 - \mu_{\tilde{A}_{S_{11}}}^2 \mu_{\tilde{A}_{S_{12}}}^2\right)^{1/2}, v_{\tilde{A}_{S_{11}}} v_{\tilde{A}_{S_{12}}}, \left(\left(1 - \mu_{\tilde{A}_{S_{12}}}^2\right)\pi_{\tilde{A}_{S_{11}}}^2 + \left(1 - \mu_{\tilde{A}_{S_{11}}}^2\right)\pi_{\tilde{A}_{S_{12}}}^2 - \pi_{\tilde{A}_{S_{11}}}^2 \pi_{\tilde{A}_{S_{12}}}^2\right)^{1/2} \right\rangle \quad (30)$$

The second way to follow is to continue without defuzzification. In this case, spherical fuzzy global preference weights are computed by using Eq. (31).

$$\prod_{j=1}^{n} \tilde{A}_{S_{ij}} = \tilde{A}_{S_{i1}} \otimes \tilde{A}_{S_{i2}} \cdots \otimes \tilde{A}_{S_{in}} \quad \forall i$$
$$\text{i.e. } \tilde{A}_{S_{11}} \otimes \tilde{A}_{S_{12}} = \left\langle \mu_{\tilde{A}_{S_{11}}} \mu_{\tilde{A}_{S_{12}}}, \left(v_{\tilde{A}_{S_{11}}}^2 + v_{\tilde{A}_{S_{12}}}^2 - v_{\tilde{A}_{S_{11}}}^2 v_{\tilde{A}_{S_{12}}}^2\right)^{1/2}, \left(\left(1 - v_{\tilde{A}_{S_{12}}}^2\right)\pi_{\tilde{A}_{S_{11}}}^2 + \left(1 - v_{\tilde{A}_{S_{11}}}^2\right)\pi_{\tilde{A}_{S_{12}}}^2 - \pi_{\tilde{A}_{S_{11}}}^2 \pi_{\tilde{A}_{S_{12}}}^2\right)^{1/2} \right\rangle \quad (31)$$

The final score ($\tilde{F}$) is calculated by using Eq. (30).

Step 6 Defuzzify the final score of each alternative by using the score function given in Eq. (27).

Step 7 Rank the alternatives with respect to the defuzzified final scores. The largest value indicates the best alternative.

The proposed approach tends to select the best alternative whose membership degree is the largest and the non-membership degree is the smallest. A large hesitancy degree is better than a large non-membership degree with equal membership degrees in terms of a better alternative.

5 An application to renewable energy location selection

According to the research results of many scientists, Aegean Region of Turkey is the best place to develop renewable energy as a result of natural conditions. Our proposed methodology is applied to selection of site location to establish wind power farm. For this goal, mostly preferred four cities (*A*1*: Çanakkale, A*2*: Manisa, A*3*: İzmir, A*4*: Balıkesir*) are evaluated. After a comprehensive literature review, four criteria and 12 sub-criteria have been determined. Criteria are *environmental conditions* (*C*1), *economical situations* (*C*2), technological *opportunities* (*C*3), and *site characteristics* (*C*4). Figure 8 illustrates this hierarchy which consists of all criteria and sub-criteria are related to them. In this structure, while "economic situations" are a non-beneficial criterion, the rest of them are beneficial. First of all, the assessments for the criteria and sub-criteria are collected from a decision-makers group with respect to the goal, using the linguistic terms given in Table 1.

The consistency ratios of the pairwise comparison matrices are calculated based on the corresponding numerical values in classical AHP method for the linguistic scale given in Table 1. Pairwise comparisons and the obtained spherical weights ($\tilde{w}^s$) and crisp weights ($\bar{w}^s$) are given in Tables 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, and 18 together with their consistency ratios (CR).

Table 19 presents the final spherical fuzzy global weights of the alternatives with respect to the evaluation criteria based on the completely fuzzy approach as given in Eq. (31) in Step 5, while Table 20 presents the spherical fuzzy weights of the alternatives based on the partially fuzzy approach as given in Eq. (29) in Step 5. Table 21 gives the results of spherical fuzzy addition using the completely fuzzy approach, while Table 22 shows the results of SF addition using the partially fuzzy approach by utilizing Eq. (30) in Step 5.

As seen in Tables 21 and 22, both approaches give the same ranking result.

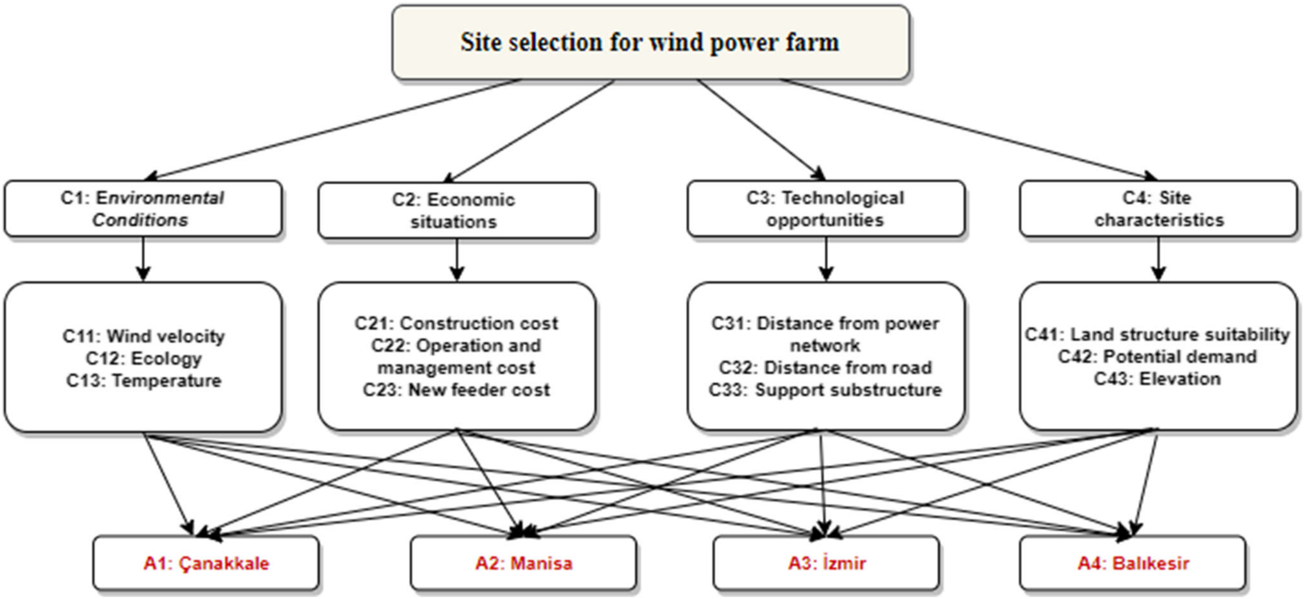


Fig. 8 Hierarchical structure for the problem

Table 3 Pairwise comparison of locations caused by environmental conditions

C1	C11	C12	C13	$\tilde{w}^s$	$\bar{w}^s$
C11	EI	VHI	HI	(0.70, 0.29, 0.24)	0.457
C12	VLI	EI	SLI	(0.39, 0.58, 0.31)	0.238
C13	LI	SMI	EI	(0.49, 0.48, 0.32)	0.305

CR = 0.057

Table 4 Pairwise comparison of locations caused by economic situations

C2	C21	C22	C23	$\tilde{w}^s$	$\bar{w}^s$
C21	EI	AMI	VHI	(0.79, 0.20, 0.18)	0.503
C22	ALI	EI	SLI	(0.38, 0.60, 0.31)	0.218
C23	VLI	SMI	EI	(0.48, 0.50, 0.31)	0.280

CR = 0.092

Table 5 Pairwise comparison of locations caused by technological opportunities

C3	C31	C32	C33	$\tilde{w}^s$	$\bar{w}^s$
C31	EI	HI	AMI	(0.76, 0.23, 0.21)	0.487
C32	LI	EI	SMI	(0.49, 0.48, 0.32)	0.292
C33	ALI	SLI	EI	(0.38, 0.60, 0.31)	0.221

CR = 0.025

Table 6 Pairwise comparison of locations caused by site characteristics

C4	C41	C42	C43	$\tilde{w}^s$	$\bar{w}^s$
C41	EI	ALI	VLI	(0.33, 0.66, 0.27)	0.180
C42	AMI	EI	SMI	(0.74, 0.25, 0.23)	0.449
C43	VHI	SLI	EI	(0.62, 0.36, 0.27)	0.371

CR = 0.070

Table 7 Pairwise comparison of alternatives caused by wind velocity

C11	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	VHI	HI	SMI	(0.67, 0.31, 0.25)	0.326
A2	VLI	EI	SLI	ALI	(0.35, 0.64, 0.28)	0.155
A3	LI	SMI	EI	LI	(0.45, 0.53, 0.30)	0.208
A4	SLI	AMI	HI	EI	(0.65, 0.35, 0.25)	0.311

CR = 0.096

Table 8 Pairwise comparison of alternatives caused by ecology

C12	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	HI	SMI	VHI	(0.67, 0.31, 0.25)	0.327
A2	LI	EI	SLI	SMI	(0.47, 0.51, 0.32)	0.216
A3	SLI	SMI	EI	VHI	(0.62, 0.37, 0.28)	0.296
A4	VLI	SLI	VLI	EI	(0.36, 0.63, 0.28)	0.160

CR = 0.052

Table 9 Pairwise comparison of alternatives caused by temperature

C13	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	SLI	ALI	ALI	(0.34, 0.66, 0.27)	0.142
A2	SMI	EI	SLI	LI	(0.47, 0.51, 0.32)	0.204
A3	AMI	SMI	EI	SMI	(0.71, 0.28, 0.25)	0.327
A4	AMI	HI	SLI	EI	(0.71, 0.29, 0.23)	0.327

CR = 0.098

Table 10 Pairwise comparison of alternatives caused by construction cost

C21	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	SMI	AMI	VHI	(0.76, 0.24, 0.206)	0.367
A2	SLI	EI	SMI	HI	(0.57, 0.41, 0.30)	0.264
A3	ALI	SLI	EI	SLI	(0.39, 0.60, 0.31)	0.170
A4	VLI	LI	SMI	EI	(0.44, 0.55, 0.29)	0.199

CR = 0.098

Table 11 Pairwise comparison of alternatives caused by operation and management costs

C22	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	SMI	VHI	SMI	(0.65, 0.34, 0.28)	0.304
A2	SLI	EI	SMI	SLI	(0.49, 0.49, 0.33)	0.217
A3	VLI	SLI	EI	VLI	(0.36, 0.63, 0.28)	0.156
A4	SLI	SMI	VHI	EI	(0.68, 0.31, 0.23)	0.324

CR = 0.060

Table 12 Pairwise comparison of alternatives caused by new feeder cost

C23	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	HI	AMI	SMI	(0.73, 0.26, 0.23)	0.347
A2	LI	EI	SMI	LI	(0.45, 0.53, 0.30)	0.202
A3	ALI	SLI	EI	VLI	(0.35, 0.64, 0.28)	0.150
A4	SLI	HI	VHI	EI	(0.65, 0.35, 0.25)	0.301

CR = 0.066

Table 13 Pairwise comparison of alternatives caused by distance from power network

C31	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	SMI	VLI	SLI	(0.46, 0.53, 0.31)	0.206
A2	SLI	EI	ALI	LI	(0.36, 0.62, 0.28)	0.160
A3	VHI	AMI	EI	HI	(0.77, 0.22, 0.18)	0.376
A4	SMI	HI	LI	EI	(0.56, 0.43, 0.29)	0.258

CR = 0.064

Table 14 Pairwise comparison of alternatives caused by distance from road

C32	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	SLI	ALI	ALI	(0.34, 0.66, 0.27)	0.142
A2	SMI	EI	SLI	LI	(0.47, 0.51, 0.32)	0.204
A3	AMI	SMI	EI	SMI	(0.71, 0.28, 0.25)	0.327
A4	AMI	HI	SLI	EI	(0.71, 0.29, 0.23)	0.327

CR = 0.098

Table 15 Pairwise comparison of alternatives caused by support structure

C33	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	SMI	VHI	SMI	(0.65, 0.34, 0.27)	0.304
A2	SLI	EI	SMI	SLI	(0.49, 0.49, 0.33)	0.217
A3	VLI	SLI	EI	VLI	(0.36, 0.63, 0.28)	0.156
A4	SLI	SMI	VHI	EI	(0.68, 0.31, 0.23)	0.324

CR = 0.060

Table 16 Pairwise comparison of alternatives caused by land structure suitability

C41	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	HI	AMI	SMI	(0.73, 0.26, 0.23)	0.347
A2	LI	EI	SMI	LI	(0.45, 0.53, 0.30)	0.202
A3	ALI	SLI	EI	VLI	(0.35, 0.64, 0.28)	0.150
A4	SLI	HI	VHI	EI	(0.65, 0.35, 0.25)	0.301

CR = 0.066

Table 17 Pairwise comparison of alternatives caused by potential demand

C42	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	SMI	VLI	SLI	(0.46, 0.53, 0.31)	0.206
A2	SLI	EI	ALI	LI	(0.36, 0.62, 0.29)	0.160
A3	VHI	AMI	EI	HI	(0.77, 0.22, 0.18)	0.376
A4	SMI	HI	LI	EI	(0.56, 0.43, 0.29)	0.258

CR = 0.064

Table 18 Pairwise comparison of alternatives caused by elevation

C43	A1	A2	A3	A4	$\tilde{w}^s$	$\bar{w}^s$
A1	EI	LI	SLI	ALI	(0.36, 0.62, 0.28)	0.160
A2	HI	EI	SMI	LI	(0.56, 0.43, 0.29)	0.258
A3	SMI	SLI	EI	VLI	(0.46, 0.53, 0.31)	0.206
A4	AMI	HI	VHI	EI	(0.77, 0.22, 0.18)	0.376

CR = 0.064

Table 19 Spherical fuzzy weight matrix based on completely fuzzy approach

Alternatives	C11	C12	C13	C21	C22	C23	C31	C32	C33	C41	C42	C43
A1	(0.34, 0.48, 0.37)	(0.19, 0.66, 0.37)	(0.12, 0.77, 0.34)	(0.21, 0.67, 0.32)	(0.09, 0.81, 0.33)	(0.12, 0.76, 0.34)	(0.13, 0.75, 0.35)	(0.06, 0.85, 0.31)	(009, 0.80, 0.34)	(0.15, 0.73, 0.33)	(0.22, 0.64, 0.38)	(0.15, 0.73, 0.35)
A2	(0.18, 0.71, 0.34)	(0.14, 0.73, 0.37)	(0.17, 0.69, 0.39)	(0.16, 0.72, 0.34)	(0.06, 0.84, 0.32)	(0.08, 0.82, 0.33)	(0.10, 0.80, 0.32)	(0.09, 0.80, 0.35)	(0.07, 0.83, 0.34)	(0.10, 0.80, 0.32)	(0.17, 0.70, 0.35)	(0.23, 0.61, 0.38)
A3	(0.23, 0.62, 0.37)	(0.18, 0.68, 0.38)	(0.26, 0.59, 0.40)	(0.11, 0.79, 0.32)	(0.05, 0.87, 0.29)	(0.06, 0.86, 0.30)	(0.22, 0.66, 0.34)	(0.13, 0.74, 0.37)	(0.05, 0.87, 0.30)	(0.07, 0.84, 0.29)	(0.37, 0.47, 0.35)	(0.19, 0.67, 0.38)
A4	(0.33, 0.50, 0.37)	(0.10, 0.79, 0.33)	(0.25, 0.59, 0.39)	(0.12, 0.77, 0.32)	(0.09, 0.81, 0.32)	(0.11, 0.78, 0.34)	(0.16, 0.71, 0.35)	(0.13, 0.75, 0.36)	(0.10, 0.80, 0.33)	(0.14, 0.75, 0.33)	(0.27, 0.57, 0.38)	(0.31, 0.52, 036)

Table 20 Spherical fuzzy weight matrix obtained from partially fuzzy approach

Alternatives	C11	C12	C13	C21	C22	C23	C31	C32	C33	C41	C42	C43
A1	(0.31, 0.83, 0.14)	(0.23, 0.91, 0.10)	(0.11, 0.96, 0.10)	(0.26, 0.89, 0.09)	(0.14, 0.96, 0.07)	(0.18, 0.94, 0.07)	(0.14, 0.95, 0.10)	(0.08, 0.98, 0.07)	(0.14, 0.96, 0.07)	(0.21, 0.93, 0.08)	(0.18, 0.91, 0.13)	(0.13, 0.95, 0.11)
A2	(0.14, 0.93, 0.12)	(0.15, 0.94, 0.11)	(0.16, 0.93, 0.12)	(0.18, 0.93, 0.11)	(0.10, 0.98, 0.07)	(0.10, 0.97, 0.07)	(0.11, 0.96, 0.09)	(0.11, 0.97, 0.08)	(0.10, 0.97, 0.08)	(0.11, 0.96, 0.08)	(0.14, 0.94, 0.12)	(0.21, 0.91, 0.12)
A3	(0.19, 0.90, 0.14)	(0.20, 0.92, 0.10)	(0.27, 0.87, 0.12)	(0.11, 0.96, 0.10)	(0.07, 0.98, 0.06)	(0.08, 0.98, 0.06)	(0.27, 0.88, 0.08)	(0.19, 0.94, 0.08)	(0.07, 0.98, 0.06)	(0.08, 0.98, 0.07)	(0.35, 0.81, 0.10)	(0.16, 0.93, 0.12)
A4	(0.29, 0.84, 0.13)	(0.11, 0.96, 0.09)	(0.27, 0.87, 0.10)	(0.13, 0.95, 0.09)	(0.15, 0.96, 0.06)	(0.15, 0.95, 0.07)	(0.17, 0.93, 0.10)	(0.18, 0.94, 0.07)	(0.15, 0.96, 0.06)	(0.17, 0.94, 0.08)	(0.23, 0.89, 0.13)	(0.32, 0.84, 0.10)

Table 21 Score values and ranking obtained from completely fuzzy approach

Alternatives	Total	Score value	Ranking
A1	(0.56, 0.02, 0.74)	10.746	3
A2	(0.45, 0.03, 0.79)	5.626	4
A3	(0.59, 0.02, 0.72)	12.226	2
A4	(0.61, 0.01, 0.71)	13.133	1

Table 22 Score values and ranking obtained from partially fuzzy approach

Alternatives	Total	Score value	Ranking
A1	(0.59, 0.41, 0.27)	16.299	3
A2	(0.45, 0.53, 0.30)	12.134	4
A3	(0.61, 0.40, 0.26)	16.885	2
A4	(0.64, 0.36, 0.25)	17.802	1

6 Comparative analysis

Kahraman et al. (2018) extended Buckley's fuzzy AHP using interval neutrosophic sets. Neutrosophic AHP has been proposed in their paper and applied to the performance comparison of law firms successfully. In this study, our proposed methodology compared with neutrosophic AHP for the site selection of wind power farm.

Table 23 presents the neutrosophic linguistic scale, which we use for the comparison purpose. In the proposed scale, we modified "absolutely more importance" degrees to ease of geometric operations.

In this comparison, the same pairwise comparisons are given in Tables 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 and 18.

We checked the consistency of pairwise comparison matrices by employing Eq. (32).

$$\mathrm{dv} = 0.6 + 0.4T - 0.2I - 0.4F \tag{32}$$

Table 23 Neutrosophic AHP linguistic scale (Kahraman et al. 2018)

Linguistic terms	(T, I, F)
Absolutely more importance (AMI)	(1, 0.07, 0.015)
Very high importance (VHI)	(0.9, 0.2, 0.1)
High importance (HI)	(0.8, 0.3, 0.2)
Slightly more importance (SMI)	(0.7, 0.4, 0.3)
Equally importance (EI)	(1, 1, 1)
Slightly low importance (SLI)	(0.02, 0.226, 0.623)
Low importance (LI)	(0.016, 0.145, 0.679)
Very low importance (VLI)	(0.013, 0.100, 0.711)
Absolutely low importance (ALI)	(0.009, 0.005, 0.765)

Table 24 Geometric means of the criteria

Criteria	Geometric mean		
	T	I	F
C1	0.87	0.30	0.17
C2	0.12	0.30	0.61
C3	0.04	0.10	0.76
C4	0.35	0.34	0.33
Total	1.37	1.05	1.88

where dv is the deneutrosophicated value. Since there is a consensus in decision-makers group, any aggregation operation is not required for our problem.

For the next step, geometric mean for criteria and sub-criteria will be calculated based on Eqs. (33), (34), and (35).

$$\begin{aligned} T_1 &= [1 \times T_{12} \times \cdots \times T_{1n}]^{1/n} \\ &\ldots \\ T_n &= [T_{n1} \times T_{n2} \times \cdots \times 1]^{1/n} \end{aligned} \tag{33}$$

$$\begin{aligned} I_{1m} &= [1 \times I_{12m} \times \cdots \times I_{1nm}]^{1/n} \\ &\ldots \\ I_{nm} &= [I_{n1m} \times I_{n2m} \times \cdots \times 1]^{1/n} \end{aligned} \tag{34}$$

$$\begin{aligned} F_{1m} &= [1 \times F_{12u} \times \cdots \times F_{1nu}]^{1/n} \\ &\ldots \\ F_{im} &= [F_{n1m} \times F_{n2m} \times \cdots \times 1]^{1/n} \end{aligned} \tag{35}$$

The result of these equations for the criteria is given in Table 24. Because of the space constraints, we do not give the rest of the geometric means of the sub-criteria and alternatives.

We obtained neutrosophic weights of criteria and sub-criteria by dividing T, I, F values with the sum of the geometric means in the row for lower, medium, and upper parameters as given in Eq. (36). Suppose that the sums of the geometric mean values in the row are a_{1s} for lower parameters, a_{2s} for medium parameters, and a_{3s} for upper parameters.

$$\tilde{r}_{ij} = \left\{ \begin{array}{l} \left(\frac{a_{1l}}{a_{3s}}, \frac{b_{1m}}{a_{2s}}, \frac{c_{1u}}{a_{1s}} \right) \\ \left(\frac{a_{2l}}{a_{3s}}, \frac{b_{2m}}{a_{2s}}, \frac{c_{2u}}{a_{1s}} \right) \\ \vdots \\ \left(\frac{a_{il}}{a_{3s}}, \frac{b_{im}}{a_{2s}}, \frac{c_{iu}}{a_{1s}} \right) \end{array} \right\} \tag{36}$$

The result of Eqs. (32) and (36) for the criteria is given in Table 25.

By using Eq. (32), we defuzzified and normalized the neutrosophic weights of criteria with the performance scores of the alternatives and calculated the overall performance score to aggregate them based on Eq. (37) as follows:

$$\bar{U}_i = \sum_{j=1}^{n} \bar{w}_j \bar{r}_{ij}, \quad \forall i \tag{37}$$

Table 25 Geometric means of the criteria

Criteria	$\tilde{r}_{ij}$			Deneutrosophicated value	Normalized value
	T	I	F		
C1	0.46	0.29	0.13	0.68	0.35
C2	0.06	0.29	0.45	0.39	0.20
C3	0.02	0.10	0.56	0.37	0.19
C4	0.18	0.33	0.25	0.51	0.26

Table 26 Local and global weights of each sub-criterion

	C11	C12	C13	C21	C22	C23	C31	C32	C33	C41	C42	C43
Local weights	0.49	0.23	0.29	0.52	0.23	0.25	0.51	0.26	0.23	0.22	0.48	0.30
Global weights	0.17	0.08	0.10	0.10	0.05	0.05	0.10	0.05	0.04	0.06	0.13	0.08

Table 27 Neutrosophic performance scores of each alternative

Alternatives	C11	C12	C13	C21	C22	C23	C31	C32	C33	C41	C42	C43
A1	0.32	0.33	0.19	0.36	0.33	0.35	0.20	0.19	0.33	0.35	0.20	0.19
A2	0.19	0.21	0.19	0.24	0.21	0.20	0.19	0.19	0.21	0.20	0.19	0.24
A3	0.20	0.27	0.34	0.19	0.19	0.19	0.37	0.34	0.19	0.19	0.37	0.20
A4	0.29	0.19	0.29	0.21	0.27	0.26	0.24	0.29	0.27	0.26	0.24	0.37

Table 28 Overall performance scores of each alternative and ranking

Overall performance score ($\bar{U}_i$)	Ranking
0.273	1
0.204	4
0.259	3
0.263	2

Table 26 indicates the local and global weights of each sub-criterion.

Table 27 shows neutrosophic performance scores of each alternative based on Eqs. (32), (33), (34), (35), and (36).

The result of Eq. (37) for each alternative is given in Table 28.

When compared with SF-AHP, the ranking has changed. The new ranking is $A_1 > A_4 > A_3 > A_2$ with NS-AHP, while it is $A_4 > A_3 > A_1 > A_2$ with SF-AHP. The main reason of this difference is the assumptions of both theories.

7 Conclusion

Three-dimensional membership functions have been very widespread in the recent years. IFS (intuitionistic fuzzy sets), PFS (Pythagorean fuzzy sets), and NS (neutrosophic sets) use those kinds of membership functions. Spherical fuzzy sets are an effort to provide a general view to three-dimensional fuzzy sets. We introduced the theory of spherical fuzzy sets (SFS) and their arithmetic operations in the literature together with their aggregation operators. This new type of fuzzy sets has been used in the extension of fuzzy AHP to SF-AHP.

Site selection of wind power farm problem has been successfully solved by SF-AHP and compared with NS-AHP. The ranking results in both methods are different, since different assumptions and scales are used.

For further research, we suggest SF-AHP to be compared with other extensions of MCDM methods such as IF-AHP (intuitionistic fuzzy AHP) and PF-AHP (Pythagorean fuzzy AHP). We also suggest spherical fuzzy preference relations (SFPRs) and inter-valued spherical fuzzy sets to be used in SF-AHP. Spherical fuzzy sets still need to be developed by using several algebraic operations including differential equations (Arqub et al. 2016; Arqub 2016; Arqub et al. 2017; Arqub 2017).

Compliance with ethical standards

Conflict of interest Author declares that he has no conflict of interest.

Human and animal rights This article does not contain any studies with human participants or animals performed by the authors.

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