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Distortions In The Flux Distribution In Two Dimensional Geometries

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Abstract

The subject of this study is distortions of flux distributions observed in Discrete Ordinates Method. To observe these distortions an important two dimensional transport code has been chosen and executed for a wellknown test problem. The distributions of scalar flux have been examined by using standard Gauss quadratures for integral terms appearing in the source term. Then, the results have been compared with the other transport code and the ray-effect problem has been discussed in detail.

Özet

Bu çalışmanın konusu, Kesikli Ordinatlar Metodunda gözlenen akı dağılımındaki bozulmalardır. Bu bozulmaları gözlemek üzere iki boyutlu önemli bir transport kodu seçilmiş ve iyi bilinen bir test problemi için koşulmuştur. Skaler akı bozulmaları, kaynak teriminde ortaya çıkan integral terimi yerine standart Gauss quadratür yaklaşımı kullanılarak incelenmiştir. Sonra, sonuçlar bir başka transport kodu ile karşılaştırılmış ve ışın-etkisi problemi ayrıntılı olarak tartışılmıştır.

Key words: *Discrete Ordinate Method, Ray-effect distortions, Gauss-Legendre quadrature*

1. Introduction

The main problem of reactor physics is the determination of neutron distribution in a system; such as a nuclear core. To determine the distribution of neutrons, the process of neutron transport which is given by neutron transport equation (Eg.1) is investigated.

$$\frac{1}{v}\frac{\partial\phi(\vec{r},E,\vec{\Omega},t)}{\partial t}+\vec{\Omega}.\vec{\nabla}\phi(\vec{r},E,\vec{\Omega},t)+\sum\nolimits_t(\vec{r},E)\phi(\vec{r},E,\vec{\Omega},t)=$$

$$\int_{4\pi} d\vec{\Omega}'\int_0^{\infty} dE'\sum\nolimits_s(\vec{r},E'\to E,\vec{\Omega}'\to\vec{\Omega})\phi(\vec{r},E',\vec{\Omega}',t)+\frac{\chi(E)}{4\pi}\int_{4\pi} d\vec{\Omega}' \qquad (1)$$

$$\int_0^{\infty} dE'\nu(E')\sum\nolimits_f(\vec{r},E')\phi(\vec{r},E',\vec{\Omega}',t)+Q(\vec{r},E,\vec{\Omega},t)$$

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The solution of transport equation plays an important role in a nuclear reactor design and analysis. This solution consists of complete distribution of particles throughout the space, energy, direction of motion and time portions of the problem. However, it is very difficult to solve this equation except for simple problems, since it has an integro-differential form with seven variables. Therefore, the equation together with the appropiate boudary conditions is solved usually by numerical methods [1,2]. These methods may be probabilistic methods (i.e. Monte Carlo) or deterministic methods (i.e.P_N-spherical harmonics expansion method and S_N-discrete ordinates method).

2. The Method

Discrete Ordinates Method, S_N, used in this study is one of the important deterministic solution methods of neutron transport equation. In the method, a set of discrete directions (or ordinates) for anqular variable is chosen and transport equation is evaluated for these directions by suitable averaging processes. The derivatives appearing in the equation are replaced by a corresponding discrete representation by using one of the finite difference techniques and the integrals in the equation are replaced by a suitable numerical integration schemes (i.e. standard Gauss quadratures [3]: Gauss – Legendre (Eg.2) and Gauss – Chebyshev (Eg.3)).

$$\int_{-1}^{+1} F(x)dx = \sum_{i=0}^{n} w_i F(x_i) \quad (2)$$

$$\int_{-1}^{+1} \frac{1}{\sqrt{1-x^2}} F(x)dx = \sum_{i=0}^{n} w_i F(x_i) \quad (3)$$

Where, w_i are called weights of the quadrature.

In this study, the angular flux (Eg.4) and the scattering transfer probability (Eg.5) are expanded in a series of Legendre polynomials [3] as,

$$\phi = \sum_{n=0}^{\infty} (2n+1) \sum_{k=0}^{n} R_n^k \phi_n^k \quad (4)$$

$$\Sigma_s = \sum_{n=0}^{ISCT} (2n+1)\sigma_{sn}(E' \rightarrow E) \sum_{k=0}^{n} R_n^k(\mu,\varphi)\phi_n^k \quad (5)$$

And, the expansion coefficients of flux are given by,

$$\phi_n^k = \int_{-1}^{+1} d\mu \int_{0}^{\pi} d\varphi R_n^k \phi \quad (6)$$

This integral, according to the Gauss-Legendre quadrature used in this study is approximated by the sum,

$$N_{nij}^k = \sum_{m=1}^{MT} w_m R_n^k(\mu_m, \phi_m) N_{ijm} \quad (7)$$

Where,

IGM : number of groups used

ISCT : scattering order

MT : number of directions used

Although the S_N method has some advantages to solve transport equation, sometimes, some unexpected problems have been met, such as ray-effect. The ray effect refers to the anomalous scalar flux distortions that appear when discrete ordinate method is applied to certain physical problems having strong absorbers and localized neutron sources in two and three dimensional geometries.

Nowadays, several remedies for eliminating or mitigating this problem have been proposed [4,5]. It seems that the practical remedy for this problem is to increase the order of S_N method (the number of directions used). In general when this is done, the frequency of oscillations becomes higher and the magnitude becomes smaller. But, it is observed that, ray-effects tend to be remarkably prersistent even for high-order S_N approximations (i.e. even when 144 directions are employed) [6,7,8].

3. The Resluts and Conclusion

In this study, to demonstrate the ray-effect distortions in scalar flux distributions one of the important transport codes, TWOTRAN-II [9] has been chosen and executed for a wellknown test problem. The problem is given in (x-y) two dimensional geometry. In this problem there is a flat isotropic source at the left bottom corner of a square region. The boundary conditions have been given as reflective on sides as seen in the figure (Fig.1). The problem data is given in the table 1.

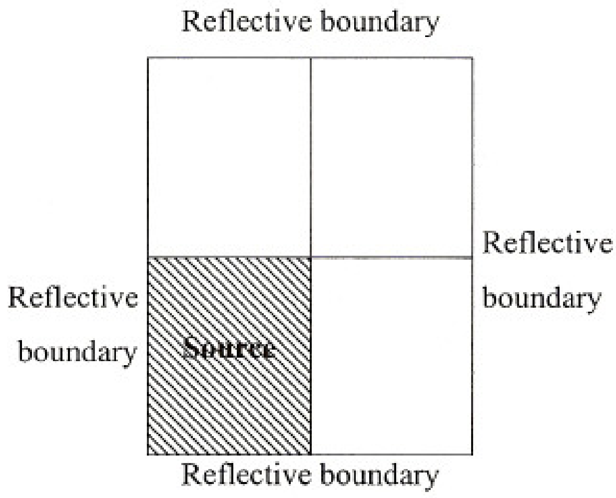

Fig.1. Sample problem

Table 1. Data for sample problem

Cross-sections	
Σ_a	0.25
Σ_s	0.50
Σ_f	0.00
Σ_t	0.75

The code has been executed for this problem and the distributions of scalar flux have been observed for $S_2 - S_{16}$ approximations (Fig.2). It is observed that, the S_2 solution is qualitatively incorrect (Fig.3). According to the Fig.4, this conclusion seems to be in agreement with the literature value (Fig.4). Because in the S_2 approximation, the number of directions taken into account is inefficient to represent the continuous distribution of flux [10]. Therefore, the solution of this approximation has not been given in the Fig.2 and not been taken in discussion. However, the higher order S_N calculations more closely exhibit the expected behavior. The S_{16} graph seems to be the best among the other approximations. The figure given below, Fig.5, also represents the comparison of TWOTRAN-II with another wellknown code, FENT. It is seen that the solution of TWOTRAN-II are closer to the reference value and exhibits less ray-effect than the solution of FENT.

At the end of this study, it is concluded that the distributions of flux obtained by this study are in agreement with the literature values. However, the study mentioned here has not been completed yet. For further studies, we are planning to examine other parameters about ray-effect and give some remedies to mitigate this problem.

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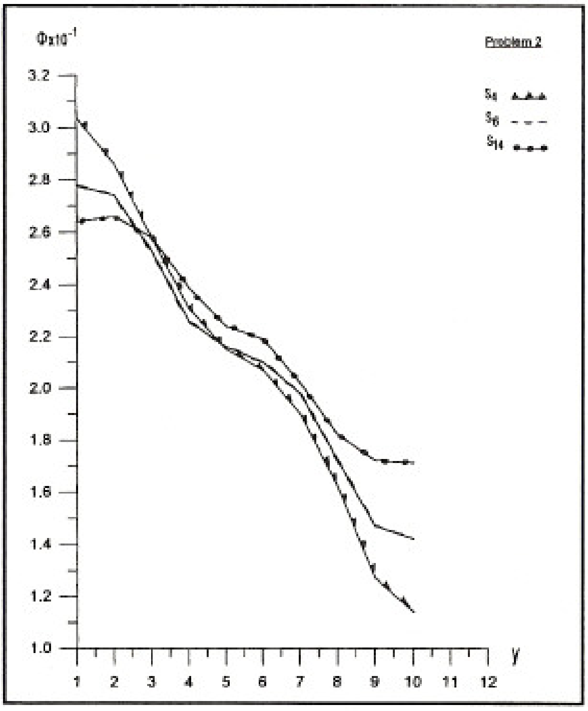

Fig.2. Distribution of flux for sample problem

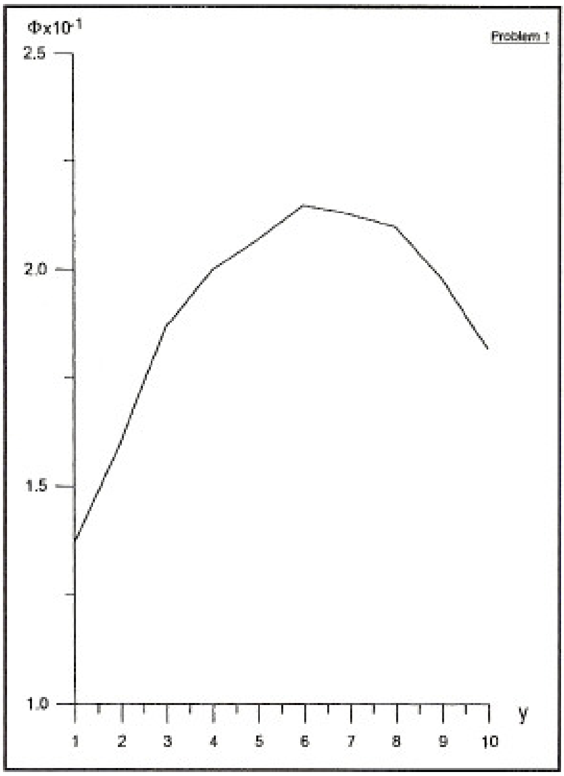

Fig.3. The S_2 approyimation for sample problem

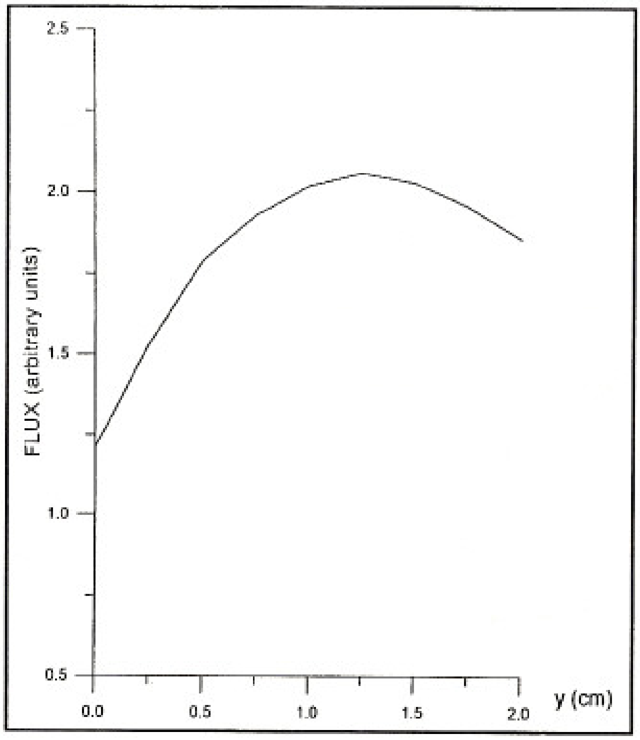

Fig.4. The S_2 distribution in the literature

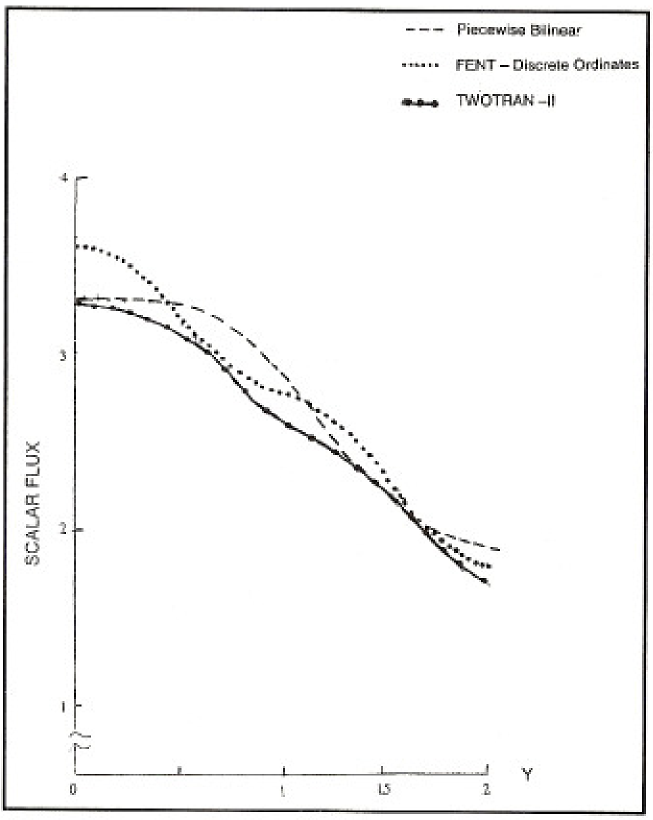

Fig.5. Comparison of results of TWOTRAN-II and FENT for sample problem

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