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A Coefficient Inequality for Convex Functions

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Abstract
In this study an important result of the paper called' A characterization for convex functions of complex order'(Ist. Üniv. Fen Fak. Matematik Dergisi cilt 54 sayfa 175- 179, 1997) is given and we present a coefficient inequality for convex functions under the regularly univalent conditions.

Özet
Biz bu makalede'A characterization for convex functions of complex order (Ist. Üniv. Fen Fak. Matematik Dergisi cilt 54 sayfa 175-170, 1997) adlı makalenin çok önemli bir neticesi olan katsayı eşitsizliğini veririz.

Keywords : Coefficient inequality, λ -Spirallike functions,Convex function of complex order.

Introduction:

Let R denote the class of functions

$$f(z) = z + a_2 z^2 + a_3 z^3 + \ldots.$$

which are analytic in the unit disc $D = \{z \ / \ |z| \leq 1\}$

A function $f(z)$ in R, is said to be a convex function of complex order b ($b \neq 0$,complex)that is $f(z) \in C(b)$ if and only if $f'(z) \neq 0$, and

$$\text{Re } (1 + \frac{1}{b} z.\frac{f''(z)}{f'(z)}) \geq 0, z \in D$$

The class $C(b)$ was introduced by P.Wiatrowski [3]. By giving specific values to b , we obtain the following important subclasses:

(i) $C(1)$ is a well known class of convex functions,

(ii) $C(1-\beta)$, $0 \leq \beta \leq 1$ is the class of convex functions of order β,

(iii) $C(e^{-i.\lambda}.Coc\lambda).|\lambda| \leq \frac{\pi}{2}$ is the class of functions for whicw $z.f'(z)$ i λ Spirallike,

(iv) $C((1-\beta).e^{-i.\lambda}.Cos\lambda), 0 \leq \beta < 1, |\lambda| < \frac{\pi}{2}$ is the class of functions for which $z.f'(z)$ **is** λ – Spirallike of order β $[See.1,3,4,5]$.

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Theorem 1.1.

Let

$$f(z) = z + a_2 z^2 + a_3 z^3 + \ldots$$

be analytic in D. A necessary and sufficient condition that

$$f(z) \in C(b)$$

is for each real number $k, .. -1 < k < 1$, the functions $F(k,b,z,\eta)$ defined by the equations, is

$$\textbf{(1.2)} \qquad F(k,b,z,\eta) = \left[\frac{k(f(z) - f(\eta))}{f(kz) - f(k\eta)} \right]^{\frac{1}{b}}$$

$$\textbf{(1.3)} \qquad F(k,b,0,0) = 1$$

$$\textbf{(1.4)} \qquad F(1,b,z,\eta) = \left[\frac{f(z) - f(\eta)}{z - \eta} \right]^{\frac{1}{b}}$$

analytic and subordinate to

$$P(z) = \frac{1+kz}{1+z}, .. z \in D$$

or equivalently that

$$\textbf{(1.5)} \qquad \operatorname{Re} F(k,b,z,\eta) \geq \frac{1+k}{2}, \left| \frac{1+k}{F(k,b,z,\eta)} - 1 \right| < 1$$

Definition:

Let $f(z)$ satisfies the inequality

$$\left| \frac{f(z) - f(\eta)}{z - \eta} \right| > m, m > 0, z \in D, \eta \in D$$

then $f(z)$ is called regularly in D [2].

Coefficient Inequality For Convex Function

In this section we shall give a coefficient inequality for convex function under the regularly univalent condition.

Now we consider the inequality (This inequality is dotained from the (1.5) for k=0,b=1)

$$\textbf{(2.1)} \qquad \operatorname{Re} F(0,1,z,\eta) = \operatorname{Re} \left[\frac{f(z) - f(\eta)}{z - \eta} \right] > \frac{1}{2}$$

on the other hand, the function

$$F(0,1,z,\eta)$$

is analytic and continous in D; therefore, we have

(2.2)
$$\lim_{z\leftarrow\eta} \mathrm{Re}(F(0,1,z,\eta)) = \lim_{z\to\eta}\left[\mathrm{Re}\frac{f(z)-f(\eta)}{z-\eta}\right]$$
$$= \mathrm{Re}(\lim_{z\leftarrow\eta}\frac{f(z)-f(\eta)}{z-\eta}) = \mathrm{Re}(f(z) > \frac{1}{2}$$

(2.3) $P(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots$

is analytic in D and satisfies $P(0) = 1, \mathrm{Re}\, P(z) > 0$ then $|p_n| \leq 2$. These functions are called Caratheodory functions. Considering the relations (2.2) and (2.3) together, we get

(2.4) $P(z) = 2.f(z) - 1$

from the relation (2.4) we have

(2.5) $2.n.a_n = p_n$

if we use Caratheodory inequality $|p_n| \leq 2$ in the equality (2.5), we obtain

(2.6) $|a_n| \leq \frac{1}{n}$

The inequality (2.6) is a new inequality for convex functions under the regularly univalent condition. This inequality is sharp because the function

$$f_*(z) = Log\frac{1}{z-1} = z + \frac{1}{2}z^2 + \frac{1}{3}z^3 + \ldots + \frac{1}{n}z^n + \ldots$$

is an extremal function and this function satisfies

$$\left|\frac{f_*(z) - f_*(\xi)}{z - z.\xi}\right| = \left|\frac{Log\dfrac{1-z.\xi}{1-z}}{z - z.\xi}\right| \neq 0 \quad ,|z| < 1, |\xi| < 1, z \neq \xi$$

Therefore, the condition of regularly univalent is satisfied by this function.

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